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ABSTRACT

This guide to accompany "Exploring Linear Measure," contains all of the student materials in SE 015 334 plus supplemental teacher materials. It includes a listing of terminal objectives, necessary equipment and teaching aids, and resource materials. Answers are given to all problems and suggestions and activities are presented for each section. Related documents are SE 015 334 and SE 015 336 through SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

ED 069574

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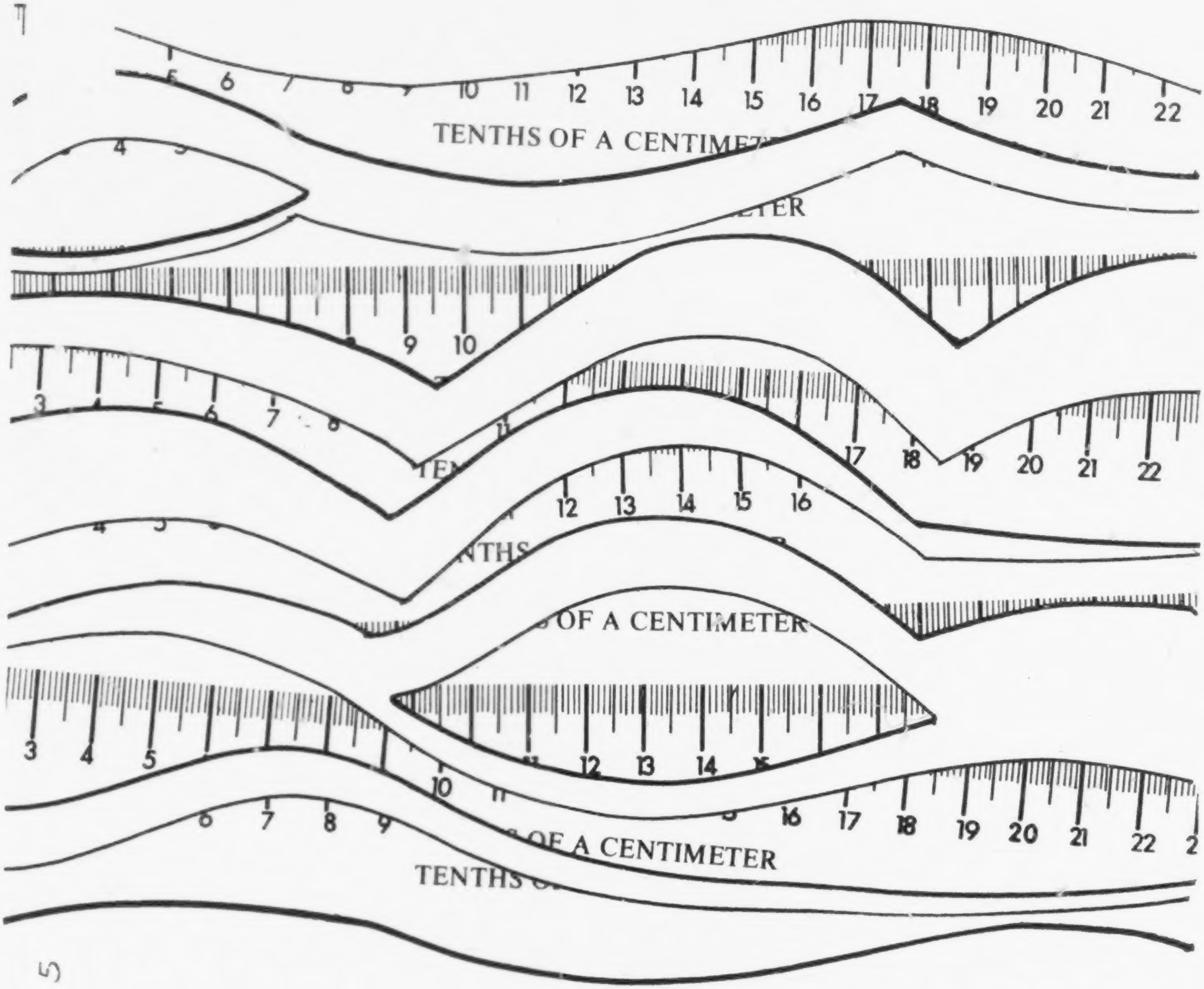
EXPLORING
LINEAR MEASURE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SE 015 335

TEACHER'S GUIDE

1

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ED 069524

EXPLORING LINEAR MEASURE

OAKLAND COUNTY MATHEMATICS PROJECT

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2100 Pontiac Lake Road
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PREFACE

BASIC ASSUMPTIONS

It is assumed that the student has not yet achieved the objectives listed for this booklet, although he has been exposed to many of the topics. Given a ruler graduated in fourths of a unit, we assume the student can measure accurately to the nearest $\frac{1}{4}$ unit. Also assumed is the ability to solve a proportion by the cross-product method (Lessons 7 and 11). In general, the student will be able to do the required computation with rational numbers. Practice on computational skills is integrated with the text material in some of the lessons (Lessons 7, 8, and 9).

OBJECTIVES

A summary of the terminal objectives for the booklet, indicating lesson(s) where each objective is developed, follows the **TABLE OF CONTENTS**. In addition, the Teacher's Guide for each lesson lists the objectives for that lesson. Although your teaching and class discussions should not be limited by these objectives, evaluation of the student's progress should be based on the stated objectives for the booklet. Use the objectives as a guide to the intent of each lesson and discuss them with your students.

The terminal objectives are also summarized for the student at the beginning of the student booklet. Indicate the location and purpose of this list to your students. As you discuss the objectives of each lesson with your students, refer them to the corresponding terminal objectives at the front of their booklet. The student can use the objectives, **✓POINT** exercises, and **EXERCISES** to evaluate his own progress.

OVERVIEW

The student is asked to function on two levels throughout the booklet. He is to: (1) exhibit his competence in the mechanics of measuring and estimating distances, and (2) to analyze data and make generalizations on the nature of measurement.

The main purpose of the booklet is for the student to develop and review the mechanics of measuring and estimating length. Both skills will be used as tools in future booklets. However, the more abstract concepts of measurement should not be neglected. Estimation activities and class discussions are given a major role in the unit.

Allow the student to experiment; to make errors without retribution; to develop his own methods of estimation; and to freely discuss his problems with both you and his classmates.

BOOKLET ORGANIZATION

In achieving these aims, different sections of the booklet serve different functions. The role for some of the major sections are as follows:

EXERCISES denotes those problems that are for supervised study in the classroom, primarily worked on an individual basis. Although it is not anticipated that much homework will be given, portions of these problems could be used as brief homework assignments.

CLASS ACTIVITY denotes those problems to be worked on a cooperative or team basis. These are not appropriate for homework. The role of the teacher is that of a resource person to be used only if needed. Allow the students to gather data and to discover on their own.

DISCUSSION QUESTIONS are an integral part of the lessons. They extend and draw together those concepts that are inherent in all measurement situations. The measurement exercises and activities will be more meaningful to the student if his results are discussed and compared with those of his classmates. These questions are to be used for in-class discussions (not homework). The Teacher's Guide for each lesson gives other discussion topics relevant to the student booklet under the headings **THINGS TO DISCUSS** and **CONTENT AND APPROACH**.

The exercises labled **✓POINT** in the student booklet (pp. 19,38,67, and 88) are to be used by the student in evaluating his own progress.

The **APPENDICES** (in Teacher's Guide only) contain enrichment activities which can be explored by individuals or groups of students to extend their knowledge of linear measure beyond the stated objectives of this booklet.

In addition to a lesson-by-lesson list of **EQUIPMENT AND TEACHING AIDS**, a summary list is given on pages x and xi of the Teacher's Guide.

Following page xix, the pages appearing on the left contain student materials. Pages 1-89 are reproduced from the student booklet. Teacher Guide pages appear on the right and are numbered, whenever possible, to correspond with the student pages. For example, page T 5 will contain comments pertaining to page 5 of the student booklet.

MEASUREMENT IS APPROXIMATE

WE CAN NOT HOPE TO ELIMINATE ALL ERRORS ---

OUR GOAL IS TO REDUCE THE SIZE AND

NUMBER OF THESE ERRORS

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SUMMARY OF TERMINAL OBJECTIVES

Lesson(s)*	
2	1. Given a measurement in caveman units, the student shall be able to give an equivalent length in feet and inches.
1,2,3	2. Given a list of caveman units, the student shall be able to rank them from shortest to longest.
3	3. Given a distance from 1" to 40', the student shall be able to give a reasonable estimate for its length (using caveman units or any other convenient estimation guides). Estimates will be checked by computing the difference between the measured distance and the estimate.
4	4. Given a list of measurements($2\frac{1}{4}$ in., $3\frac{1}{2}$ in., $6\ \frac{3}{8}$ in.,...), the student shall be able to indicate which is most precise.
5,6	5. Given a list of metric and English units of length, the student shall be able to rank them from smallest to largest.
5,6	6. The student shall be able to define the most commonly used metric prefixes.
5,6	7. The student shall be able to state the numerical relationships between the basic units within the English and the metric system.
7	8. Given a ruler graduated to the nearest $\frac{1}{16}$ in., $\frac{1}{10}$ in., or $\frac{1}{10}$ cm. and an indicated length such as 3.7 in. or $2\ \frac{5}{8}$ in., the student shall be able to indicate the corresponding location on the ruler scale.

* The lesson number(s) indicate the principal development of the objective.

Lesson(s)

8,9 9. The student shall be able to measure accurately to the nearest $\frac{1}{16}$ inch, .1 centimeter, and .1 inch using any arbitrary point on the ruler scale as the "zero point".

10,11 10. Given information such as 1 ft. = 12 in. and 4 ft. = ? in., the student shall be able to find the missing value.

5,12 11. Given a distance to be measured, the student shall be able to select a suitable English and a suitable metric unit for measuring this distance.

This booklet emphasizes accurate use of a ruler and related measurement skills.

However, some computational skills are integrated with some of the text material as a review for the student. Computational objectives reviewed in the booklet are listed below.

Using rulers graduated in subdivisions of an inch ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{16}$) as a guide, the student shall be able to answer questions such as...

7 1. If $\frac{3}{4} = \frac{x}{8}$, then x = ______.

7,8,9 2. If $\frac{1}{2} + \frac{1}{8} = \frac{y}{8}$, then y = ______.

9 3. If $\frac{3}{4} - \frac{3}{16} = \frac{a}{16}$, then a = ______.

9 4. 6.5 - 3.7 = ______.

NOTE: Individual lessons sometimes list objectives other than the fifteen listed here. In most cases, there are sub-objectives related to one of these terminal objectives.

EQUIPMENT AND TEACHING AIDS

A. STUDENT	LESSON
*1. film preview guide	0
*2. discussion questions for film	0
3. masking tape	2,3,13
4. 100 ft. (or 50 ft.) tape measure	2,3
*5. rulers	
a. whole inch	5,8
b. $\frac{1}{2}$ inch	2,3,7,8
c. $\frac{1}{4}$ inch	7,8
d. $\frac{1}{8}$ inch	7,8
e. $\frac{1}{16}$ inch (2 types)	7,8,9,13
f. .1 inch (2 types)	7,8,13
g. whole cm.	5,7,8
h. .1 cm.	7,8,9,13
i. broken ruler	
(i) .1 cm.	9
(ii) $\frac{1}{16}$ inch	9
6. yardstick	3,13
7. encyclopedia	4
8. dictionary	4
9. piece of string (about 8 ft. long)	13
10. meter stick	13

*Indicates those materials provided with the booklets.

B. TEACHER		**LESSON**
1.	16mm film, Measurement and Man	0
2.	16mm projector and take-up reel	0
3.	projection screen	0,1,2,7,8,9
4.	overhead projector	1,2,7,8,9
*5.	prepared transparencies	1,2,8
6.	overlay for checking exercises	1,8
*7.	acetate rulers (same precisions as those provided for the student)	2,7,8,9
8.	postal scale	3
9.	25-30 pennies	3
10.	one quarter	3
11.	one dollar bill	3
12.	yardstick	5
13.	meter stick	5
*14.	acetate T-square overlay	1,2,8

*Indicates those materials provided with the booklets.

NOTE: The Teacher's Guide for each lesson lists the equipment and teaching aids needed for that lesson. The asterisk (*) is used throughout the booklet to indicate materials provided with the booklets.

RESOURCE MATERIALS

The history and development of measurement systems is fascinating. The student booklet contains interspersed anecdotes on this development. While not part of the mainstream, they will be of interest to some students. (See ORIGIN OF THE MILE - p.39, RULE OF THUMB - p.12, THE GATEWAY ARCH - p.23 , HOW LONG IS AN INCH - p. 28 ,...)

There are several sources containing additional information for those wishing to explore the development and uses in greater depth than the summaries in this booklet. Some of these are listed in the following annotated bibliography.

I. HISTORY AND DEVELOPMENT OF MEASUREMENT

The Amazing Story of Measurement. Lufkin Rule Company, Saginaw, Michigan, 48601. 1968.

This pamphlet uses a comic book format to illustrate the history of linear measure. It could be used by an entire class or portions thereof, to get a good summary of this history. Copies are free upon request to Lufkin Rule Company.

"Approximately Exact", General Motors Corporation, Educational Relations Section, Detroit, Michigan, 48202. 1964.

This four-page article is Number 10 in the Mathematics at Work in General Motors series. This series of twelve articles is designed as a teaching aid for secondary mathematics teachers. Each article describes an application of mathematics to General Motors research and development.

The article concentrates on the topics of significant digits, approximate numbers, absolute error, relative error, and percentage of error -- topics not particularly applicable to this booklet. However, the first part of the article contains a concise discussion of the need for differing precisions in measurement and the approximate nature of measurement, topics which are applicable to the development of this booklet.

The Mathematics at Work series is free upon request to the above address.

Isaac Asimov, Realm of Measure. New York: Fawcett World Library, 1967.

Beginning with the use of parts of the body as units of measure, this paperback book traces the history of measurement up to the use of complex units. Several types of quantities are discussed (capacity, time, area, length, weight,...). This book is extremely readable and contains several chapters related to the material you will use in your classroom.

Jeanne Bendick, How Much and How Many. New York: McGraw - Hill, Inc., 1947.

This book, although not of recent origin, is extremely readable and gives an excellent summary of the basis for measurements in many areas (time, medicine, weather, navigation, clothing, printing,...). Also contained is a short chapter (10 pages) summarizing the function and history of the National Bureau of Standards. An excellent book for both student and teacher.

Roger Burlingame, Machines that Built America. New York: Signet Books, New American Library, 1953.

This paperback book gives an account of the machines which helped spur the development of our industrial society, the men who created them, and the economic climate that spurred their production. The dependence of this historical development upon improved methods of measurement is highlighted throughout the book.

William H. Glenn, and Donovan A. Johnson, The World of Measurement. St. Louis, Missouri: Webster Division, McGraw- Hill, 1961.

The booklet is part of the Exploring Mathematics on Your Own series found in most department libraries. The book contains a short summary of many different units of measure. It contains many questions appropriate for class discussion and ideas for class or individual measurement projects.

A History of Measurement. Ford Motor Company, Educational Affairs Department, The American Road, Dearborn, Michigan, 48121.

This chart is seen in many classrooms and should be. It is an excellent visual aid outlining the history of linear measure and should be hung in the classroom prior to and during this unit on measurement. The chart is free upon request to the above address.

Measuring Systems and Their History. Ford Motor Company, Educational Affairs Department, The American Road, Dearborn, Michigan, 48121. October 1966.

This 26-page pamphlet published by the engineering staff of Ford outlines and traces the history and development of measurement systems (length, mass and volume) from the early Egyptian and Sumerian Civilizations to the metric changeover in England and the study of a similar changeover in the United States. Chapter headings are "Civilization and Metrology", "Measurement in the Middle Ages", "Evolution of the Metric System", "Britain and the Metric System", and "Measuring Systems in the United States". A free copy may be obtained upon request to the above address.

Precision - A Measure of Progress. General Motors,
Public Relations Staff, Detroit, Michigan, 48202.

This pamphlet is more technical than most of the other selections listed, but contains many sections that would be of interest to both teacher and student. It is an excellent source for learning about recent advances in techniques of measurement and also contains well-written sections on the history of measurement. Class sets are free upon request to the above address.

Verne N. Rockcastle, Manual for Measurement Science.
Ohaus Scale Corporation, 1050 Commerce Avenue,
Union, New Jersey, 17083. 1965.

This booklet is written for grades 4 and up. It contains sections on the meaning of measurement, development of standards, activities with the ruler, how to use the vernier caliper and pan balance, and a description of 16 experiments which can be done with a balance and inexpensive materials. One copy is free upon request to the above address.

The Tools and Rules for Precision Measuring. The
L. S. Starrett Company, Athol, Massachusetts,
01331. 1965.

This booklet is designed for students in vocational schools and apprentices in industrial training classes. It illustrates the uses of several precision measuring instruments and their industrial applications. Included are the micrometer, vernier caliper, gage blocks, steel tapes, and many other miscellaneous gauges. Copies are free upon request to the above address.

The controversy as to whether or not the United States should make the metric system mandatory is a social issue which affects all of us. It is also an issue about which most persons are either unknowledgeable or misinformed. For example, few persons realize that the metric system has been the only *legal* system of measurement in the United States since 1866. However, at that time, its use was not made mandatory.

Some references on the metric system are:

II. THE METRIC SYSTEM

Valerie Antoine and John T. Milek, *Bibliography of the Metric System,* The Metric Association, Inc., 2004 Ash Street, Waukegan, Illinois, 60085. 1968.

The authors have compiled an extensive bibliography of articles and resources (a total of 1194) on the metric system and its usage. The cost is $2.25 ($1.50 for members). The Metric Association has other publications which are available at minimal cost to both members and non-members. Members receive a quarterly news letter which describes recent trends in metrication and recent sources of information. Information on publications and membership can be obtained by writing the above address.

Alden P. Armagnac, "The New Push for the Metric System: Will You Give Up Pounds, Feet, and Inches?", Popular Science, pp. 54-57, 204, June, 1969.

The author analyzes the metric changeover controversy. Of special interest is two diagrams and a description of a recently designed invention which would convert an existing machine tool (lathe or milling machine) into a metric one—at a small fraction of the cost of replacing it.

Brief History and Use of The English and Metric Systems of Measurement. NBS Special Publication 304 A. National Bureau of Standards, Washington, D.C., 1968.

In addition to outlining the history and use of the English and metric systems of measurement, this short pamphlet (4 pages) includes a small(16 "x 10½") wall chart of the modern metric system (SI). This pamphlet is available from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402, for 20 cents. Also available is a full-scale wall chart (45"x29"), NBS Special Publication 304, for 50 cents.

Ralph M. Drews and Chauncey D. Leaks, "The Metric System - Pro and Con", Popular Mechanics, pp. 138-39, December 1960.

Both sides of the metric system controversy are expressed. Each author outlines the supportive evidence for one position. This is an excellent summary of both positions.

Nolan Hudson, "Metric System", *Mathematics in Michigan*, Michigan Council of Teachers of Mathematics, No. 23: pp. 3-5, January, 1969.

This article summarizes the reasons for switching to compulsary use of the metric system and some obstacles hindering a changeover.

Harland Manchester, "Must America Go Metric?", *Reader's Digest*, pp.63-70, October, 1968.

The author discusses recent developments in the metric changeover controversy. A summary of the problems Britain is facing in the process of converting to the metric system is included.

National Council of Teachers of Mathematics. *The Metric System of Weights and Measures.* Twentieth Yearbook. New York: Bureau of Publications, Teachers College, Columbia University, 1948.

Although not of recent origin, this collection of 73 articles is probably the most comprehensive source of information supporting mandatory use of the metric system. Among the topics covered by the articles are the history of measurement, use of the metric system in various industries, and suggestions for teaching the metric system. This book is a must for anyone who is planning to teach students the use of the metric system.

NOTE: Information on the status of current legislation concerning the meteric system may be obtained by writing your U.S. senator or congressman. Senator Robert Griffin has been especially active in the area of legislation concerning the metric system.

MEASUREMENT AND MAN

OBJECTIVES

1. To discuss the need for measurement.
2. To illustrate situations using measurement.
3. To illustrate the use of several measuring instruments.
4. To illustrate the variety of units used for measurement.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

* 1. Film preview guide

* 2. Discussion Questions for film

B. TEACHER

1. 16 mm film, MEASUREMENT AND MAN
2. 16 mm projector and take-up reel
3. projection screen

MEASUREMENT AND MAN 16 mm sd. color ISC-603...............$5.90

b & w IS-603................$4.15

Develops the importance of measurement in science and in our daily lives, points out the vast variety and number of measurements in use today, and shows the descriptive and comparative nature of measurement. Shows that measurement is a very important tool in accurate communication and data recording. Stresses the importance of precision measurement in science and in industry. (Indiana U A-V Center)
Weights and measures jh sh

*Ditto masters for these sheets are provided with the text.

(This guide is reproduced on a ditto master for distribution to students prior to showing the film.)

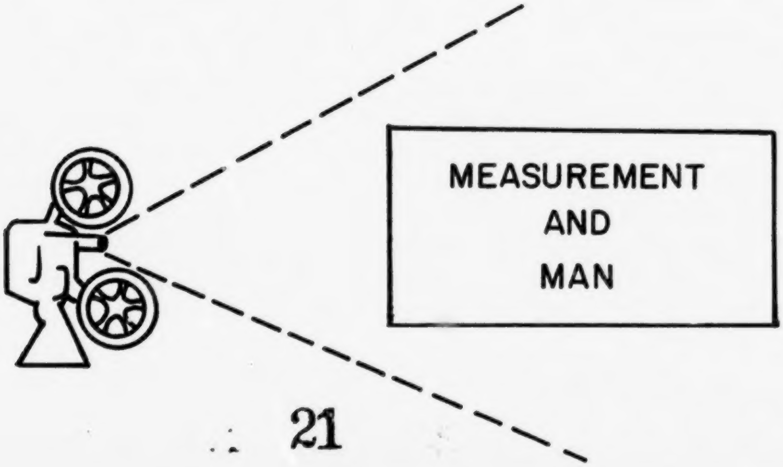
MEASUREMENT AND MAN

The film you are about to see illustrates several situations involving measurement and the use of measuring instruments. The measurements which are demonstrated use several different units of length.

Watch and listen carefully. Watch for and think about the following items as you view the film.

THINGS TO WATCH FOR AND THINK ABOUT

1. The different units of measurement used.
2. The different measuring instruments used.
3. How many of these uses of measurement are you familiar with?
4. How many times do you see measurements being used each day?
5. Compare the units of length used by early civilizations with those used today. How are they different? In what ways are they alike?
6. What is measurement?



The producers of MEASUREMENT AND MAN, the Indiana University Audio-Visual Center, give the preceding description in their 1968 catalog. The rental prices quoted are rentals for a period of 5 calendar days. Films are scheduled for two days' use unless a longer period is requested. Schools needing films more than 5 days can book them for an additional week's use or any part thereof for .6 times the catalog price. For specific information, consult their catalog. Order from Indiana University Audio-Visual Center, Bloomington, Indiana. Allow 2-3 weeks for delivery.

CONTENT AND APPROACH

This introductory lesson exposes the student to the diverse uses of measurement and measuring instruments. The discussion of the film and pages 1 and 2 of the text provides an opportunity for the teacher and student to discover in what aspects of measurement the students are interested and knowledgeable. The student should discover that measurements of many types are commonplace and familiar to him.

The discussion questions provided are not meant to be used for supervised study or homework but as a basis for exploring the measurement ideas highlighted by the film. Most of these ideas will be treated in greater detail at a later date. Thus, there is no need to belabor any one topic. What you discover about the student's background in this lesson will help determine which parts of this unit will be relatively new material and which parts will be a review and extension of previous knowledge. Listen to them!

To achieve the desired results, prior to showing the film give each student a copy of the film preview guide (See page xii.) indicating what to observe. If some set-up time is needed, the students could use this time to read the guide. After the film is shown and before it is discussed, allow the students a few minutes to read the DISCUSSION QUESTIONS (See pages xxiv and xxvi .) and jot down notes to use in the discussion.

This could be done during the time the projection equipment is being dismantled. The list of discussion questions is not exhaustive. It is meant as an example of the types of questions which can be initiated by the film.

The *first half of the film illustrates a variety of situations using measurement*: 1) timing a track meet, 2) buying shoes, 3) buying slacks, 4) baking a cake, 5) buying gas, 6) household carpentry, and 7) similar activities. The use of parts of the body as linear units by early civilizations is illustrated. The variability of these units initiates an investigation of the need for uniform standard units to facilitate communication.

The *second portion of the film concentrates on the need for and use of precise measurement by industry.* Several measuring instruments used by industry are shown in operation.

With the exception of *two flaws* which may be easily overcome, the film does an excellent job of indicating both the nature and uses of measurement. First, the word "exact" is sometimes used interchangeably with the word "precise". As a result, the approximate nature of measurement is not given its deserved emphasis. Another weakness is the almost exclusive use of the English system of measurement as opposed to the metric system. Only once is the use of a metric unit mentioned. The role of the metric system in the development of measurement and its use in our present society should not be neglected.

(These questions are reproduced on a ditto master for distribution to students following the film.)

DISCUSSION GUIDE
FOR
MEASUREMENT AND MAN

DISCUSSION QUESTIONS

1. The time for the race shown at the beginning of the film was 21.9 seconds. What was the probable distance of this race?

2. Name some of the units of measure used in the film.

3. Name some of the measuring instruments used in the film.

4. What units of length could be used instead of the feet, inches, and parts of the body mentioned in the film?

5. The film illustrated several precise measurements, using very small units of measure. How many of these were exact measurements?

6. One part of the film showed a scientist taking several different measurements (of time) for the same experiment instead of relying on one measurement. Why measure more than once? Is measuring more than once just a waste of time?

7. The film stated that measurement is done by comparison. What is the meaning of this statement?

8. The film concluded with the observation that "Wherever man is, there will be measurement." Do you agree with this statement? Why or why not?

ANSWERS

(Discussion Questions on the Film)

1. A 220 yard (or 200 meter) dash.
2. Following is a partial list of the units used.

foot
yard
inch
second
minute
cubit
span
hand
pace
thumb-width

These parts of the body were not named but were used as units of length. The text will name these later. An acceptable answer would be "parts of the body" or a description of the unit.

light-year
teaspoon
cup
gallon
degree

foot-pound
pound
mile
millimeter

3. Following is a partial list of the instruments used.

stopwatch
gas pump dial
tire gauge
micrometer
bathroom scale
yardstick
clock

thermometer
measuring cup
measuring spoon
vernier caliper
ruler
tape measure
device to measure length of cloth

4. This question may be vague. The intent is to mention the metric system.
5. None.

9. Measurement was described as the "language of accurate description." Why might this be a good definition for measurement?

10. Even though we have standard units of measure, we still use many non-standardized units. The film mentioned two such units - "as long as my arm", and "as tall as I am". Can you name other similar units or expressions that are still used today? In what kinds of situations are such units used?

11. Why are uniform units so important? Could we operate our present society without uniform standards?

6. The reason for more than one measurement is based on the approximate nature of measurement and the desire to reduce the chance of error. Some similar activities are:
    a. Judging of diving and skating events is done by several judges. The highest and lowest scores are dropped and the average of the remaining scores is multiplied by the degree of difficulty to obtain the contestant's score for that dive or figure.
    b. Track events (those not electronically timed) are usually timed by more than one person to reduce the chances for error.

7. Answers will vary.
8. Answers will vary.
9. Answers will vary.
10. Some other possibilities are:
    a. A pinch or dash of salt.
    b. I wouldn't trust him as far as I could throw him.
    c. A stone's throw.
    d. I live about ten minutes from town (using time approximations to measure distance).

    Non-standardized units such as these are usually used i everyday conversation and situations where precision is not required, only a good estimate or visual image.

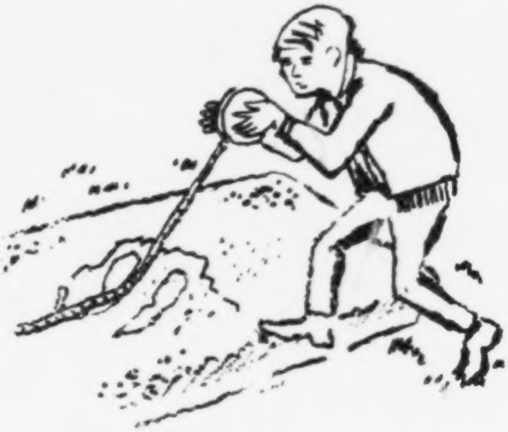
11. Answers will vary to the first question. The answer to the second question is obviously no.

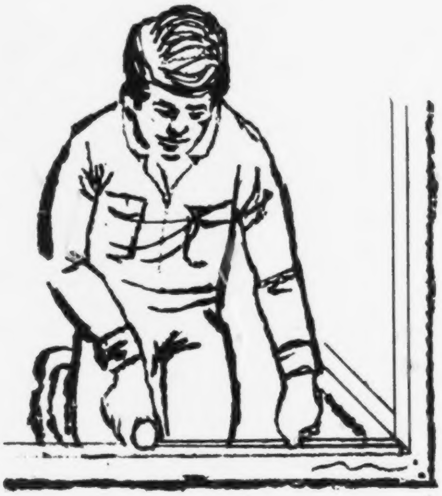
STUDENT BOOKLET
AND
TEACHING SUGGESTIONS

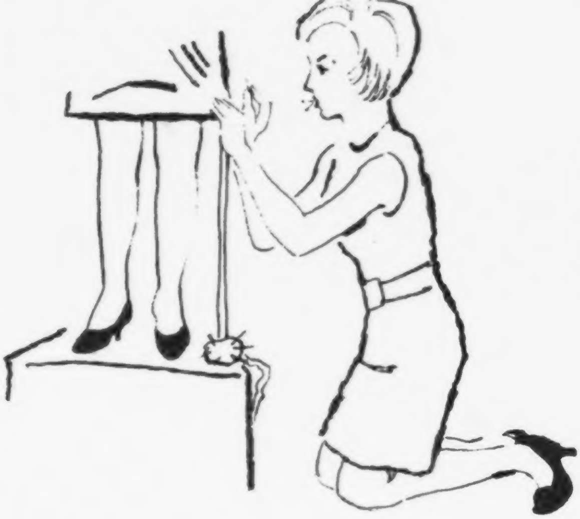
WHY MEASURE LENGTH?

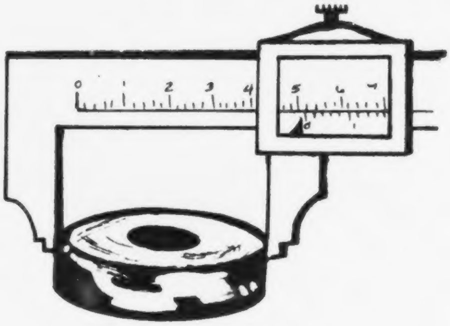
Hardly an hour goes by that you do not think about or perform some measurement. Measuring instruments are commonplace items. (Yardsticks, tape measures, speedometers, thermometers, clocks...)

This unit will concentrate upon the measuring of length. Illustrated below are some situations involving length.









WHY MEASURE LENGTH?

(These or similar questions may be reproduced on a ditto master for distribution to students if page 1 of the text is discussed.)

DISCUSSION QUESTIONS

1. What measurements are associated with each of the following activities?
    a. doing your homework
    b. pouring a cement sidewalk
    c. making a dress
    d. baking a cake
    e. dieting
2. Name a unit of measure which could be used to measure each of the following.
    a. temperature
    b. amount of gasoline used
    c. amount of electricity used
    d. distance from Detroit to Chicago
3. Different types of measurement are used in different settings. What measuring devices and units of measure are commonly used in the following locations?
    a. service station
    b. kitchen
    c. gymnasium
    d. airport
    e. clothing store
    f. drag strip

A class discussion concerning page 1 of the text is an optional activity. The above list of discussion questions illustrates the types of questions which could be used. A discussion of page 1 could be combined with the discussion of the film, Measurement and Man.

MEASUREMENT IS APPROXIMATE.

WE CAN NOT HOPE TO ELIMINATE ALL ERRORS ---

OUR GOAL IS TO REDUCE THE SIZE AND

NUMBER OF THESE ERRORS.

The statement on page 2 indicates part of the philisophy and spirit with which this booklet should be taught. It is hoped that at the end of the booklet, the student will realize (1) that measurement is approximate and (2) that through precise measurement a better approximation of the actual length is obtained.

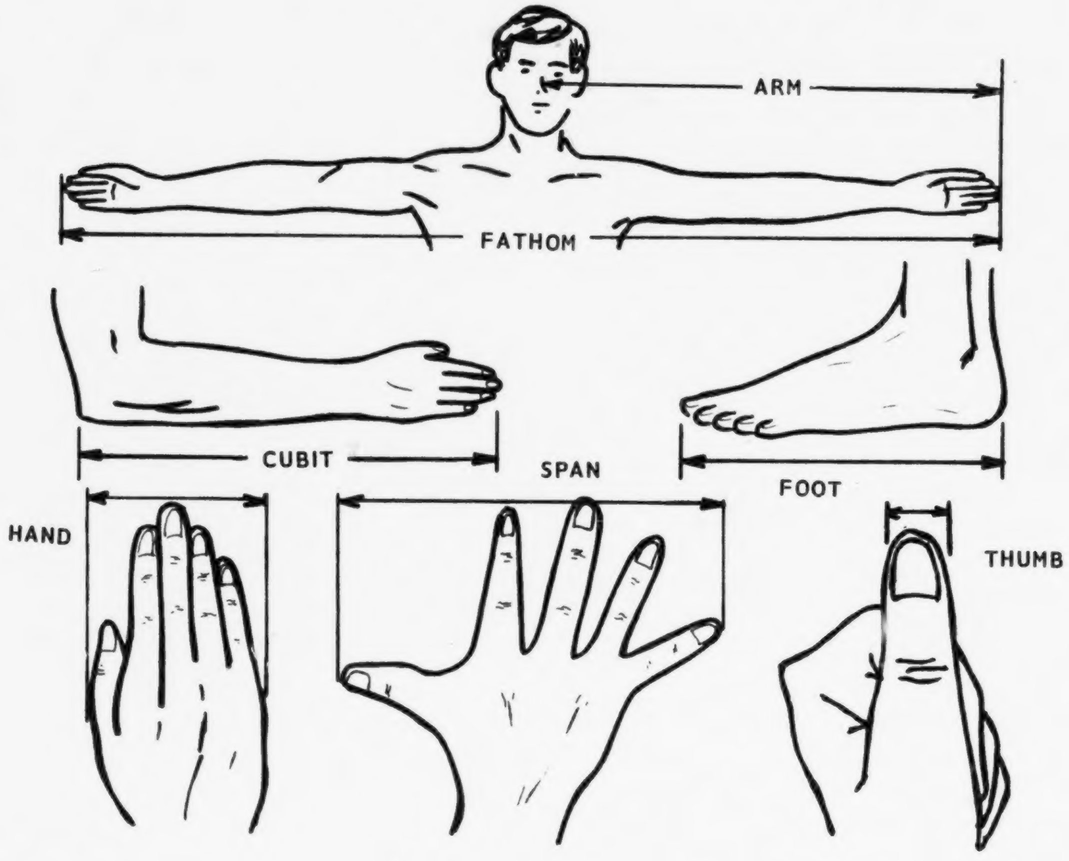
THE WALKING YARDSTICK

OUR WORLD OF MEASUREMENT . . .

Man has always measured objects around him. The illustrations on page 1 indicate some uses of measurement. You are familiar with most of these.

SINCE TIME BEGAN . . .

Some situations require accurate measurements. The caveman did not need accuracy. Any cave would do if it would hold him and his family. Distances were judged by sight or time and sizes were measured by paces or by matching the objects to be measured with stones, trees, and other objects familiar to him.



THE WALKING YARDSTICK

OBJECTIVES

1. Using parts of the body as units of length, (referred to as caveman units) the student shall measure distances in the classroom and vicinity to...
    a. demonstrate his ability to measure to the nearest unit and $\frac{1}{2}$ unit.
    b. demonstrate his knowledge of the definitions for the caveman units listed in the text.
    c. illustrate the variability of the caveman units (to be highlighted in the DISCUSSION QUESTIONS
2. Given a drawing of a unit of length (caveman, English, metric, or any arbitrary unit), the student shall be able to give a reasonable estimate of the location of a line which will divide the unit of length into two equal parts.

EQUIPMENT AND TEACHING AIDS

TEACHER

1. Overhead projector
2. Projection screen

*3. Prepared transparencies

*4. Overlay (for checking CLASS ACTIVITY I , pp. 7-8)

CONTENT AND APPROACH

Units of length used by early man are investigated for their historical value, their relationship to our present units (Lesson 2), and their value as guides for estimating distances (Lesson 3). The use of these early units and an investigation of their properties illustrates some of the basic properties of all measurement systems and units of measure (their approximate nature, the arbitrariness of units, the process of measuring length).

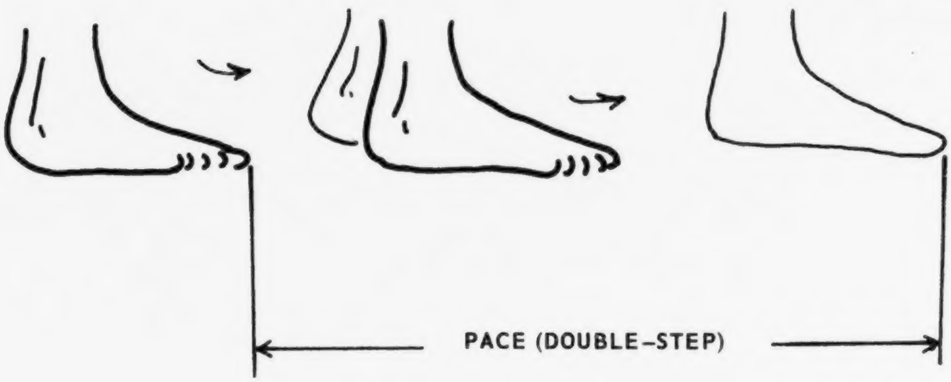
The diagrams (pages 3-4) and list (page 4) define the caveman units to be used in this booklet.

THE BODY BECOMES A YARDSTICK...

Many of the units of length used by the caveman were based on parts of the body. The human body actually served as a walking yardstick! Except as a guide in estimation, these early units are no longer widely used. However, some of our present-day units are related to them.

The following list describes some units of length related to parts of the body. (See sketches on the previous page and at the bottom of this page.)

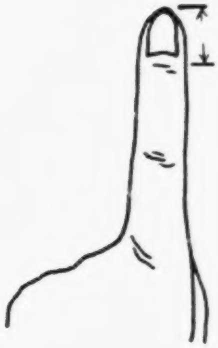
a. Width of THUMB
b. Distance from first joint to end of FOREFINGER
c. HAND - width of hand
d. SPAN - distance from end of the thumb to end of little finger when the hand is fully stretched out
e. CUBIT - length of forearm (from tip of elbow to tip of the middle finger of the outstretched arm)
f. Length of FOOT
g. Distance from nose to thumb of outstretched arm (ARM)
h. Distance across outstretched arms (FATHOM)
i. PACE - length of one double-step



CONTENT AND APPROACH

As your students work through the exercises of Lessons 1-3, they should develop immediate recall of the definitions listed on page 4. These units of length will be used throughout the booklet, especially in Lessons 1-3 and as guides for estimating lengths.

Use the diagrams on pages 3 and 4 as aids in visualizing the definitions for the units. The forefinger is not illustrated on either page 3 or 4 due to a lack of space. The forefinger is illustrated on page 8. It is defined as the distance from the first joint to the end of the forefinger (See sketch below).



FOREFINGER

MEASURING LENGTH

Given a distance to be measured...

(1) Select some length to use as a unit of length.

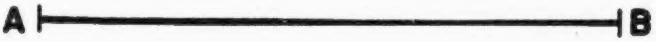
(2) "Mark off" the distance being measured in unit lengths.

(3) Count the number of unit lengths it takes to "fill up" the distance being measured.

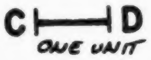
EXAMPLE 1:

Measure the length of $\overline{AB}$ (read "segment AB" to the nearest whole unit.

SOLUTION :

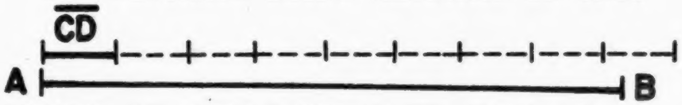


(1) SELECT SOME LENGTH TO USE AS A UNIT OF LENGTH.



Suppose $\overline{CD}$ is selected as the unit of length.

(2) "MARK OFF" THE DISTANCE BEING MEASURED IN UNIT LENGTHS.



(3) COUNT THE NUMBER OF UNIT LENGTHS IT TAKES TO "FILL UP" THE DISTANCE BEING MEASURED.

It takes more than 8 and less than 9 units to "fill up" the length of $\overline{AB}$. This is written...

$$8 \text{ units} < m(\overline{AB}) < 9 \text{ units.}$$

[$m(\overline{AB})$ is read "the measure of segment AB".]

The length of $\overline{AB}$ is closer to 8 units than 9 units. When measured to the nearest whole unit,...

$m(\overline{AB})$ is 8 units long.

CONTENT AND APPROACH

The **CLASS ACTIVITY** (pages 7-10) requires the student to...

(1) estimate the location of a line which will divide a given unit of length into two equal parts, and...

(2) use the caveman units listed on page 4 to measure to the nearest whole and $\frac{1}{2}$ unit.

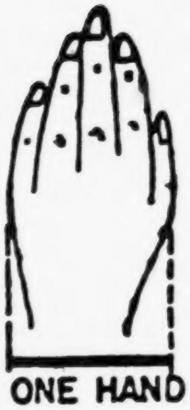
The sample solutions on pages 5-7 and use of the transparencies will illustrate the skills needed for successful completion of the **CLASS ACTIVITY**.

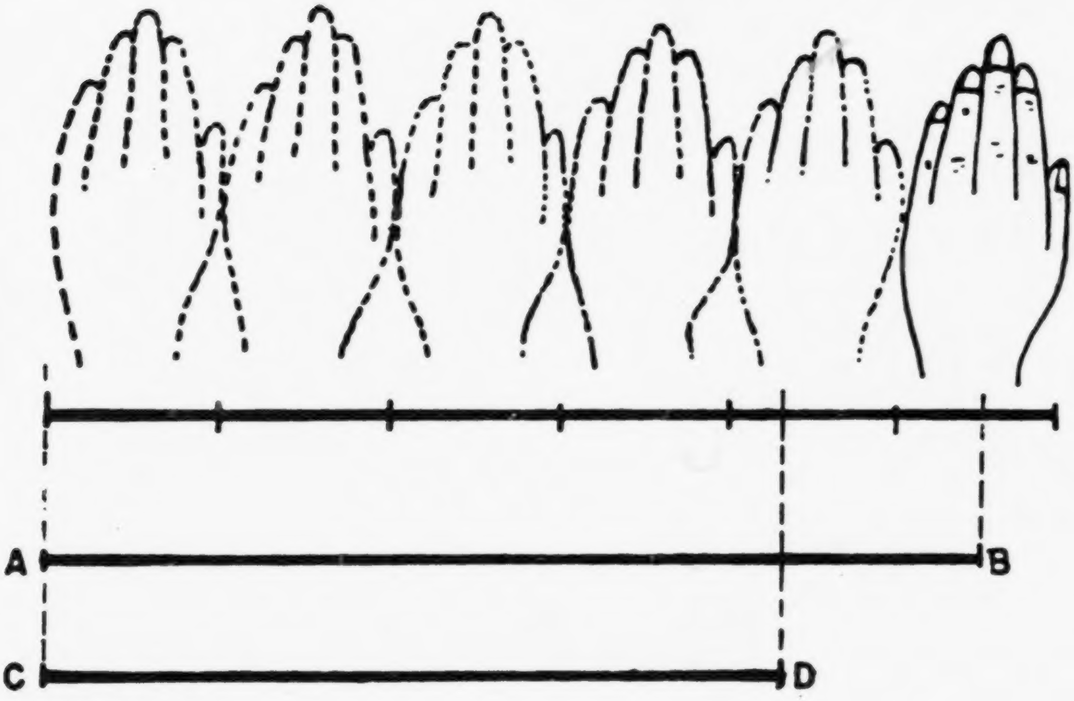
Page 5 summarizes the process of measuring length. Some discussion questions which could be used with this page are:

1. Could a unit of length other than $\overline{CD}$ have been used?
2. (a) If your answer to question #1 is yes, what effect would using a different unit have upon the length of the segment? the number used to express the length? (No effect on the length but it usually causes a change in the number used to express the length. This is an important distinction but should not be belabored at this stage.)

   (b) Would some units of length be "more suitable" than others to measure the length of $\overline{AB}$? (This concept will be developed in Lesson 12.)
3. Suppose the "marking off" of unit lengths was begun at B instead of A. Would this have any effect on the measure of the segment?

EXAMPLE 2: Measure $\overline{AB}$ and $\overline{CD}$ to the nearest hand.





SOLUTIONS :

5 hands $< m(\overline{AB}) <$ 6 hands.
To the nearest hand, $m(\overline{AB}) = 6$ hands.

4 hands $< m(\overline{CD}) <$ 5 hands.
To the nearest hand, $< m(\overline{CD}) = 4$ hands.

CONTENT AND APPROACH

Use the sample solutions on pages 6 and 7 and the first four transparencies to review the process of measuring to the nearest whole and ½ unit.

TR 1-1 through TR 1-4

Ask the students to give the measure of each segment...

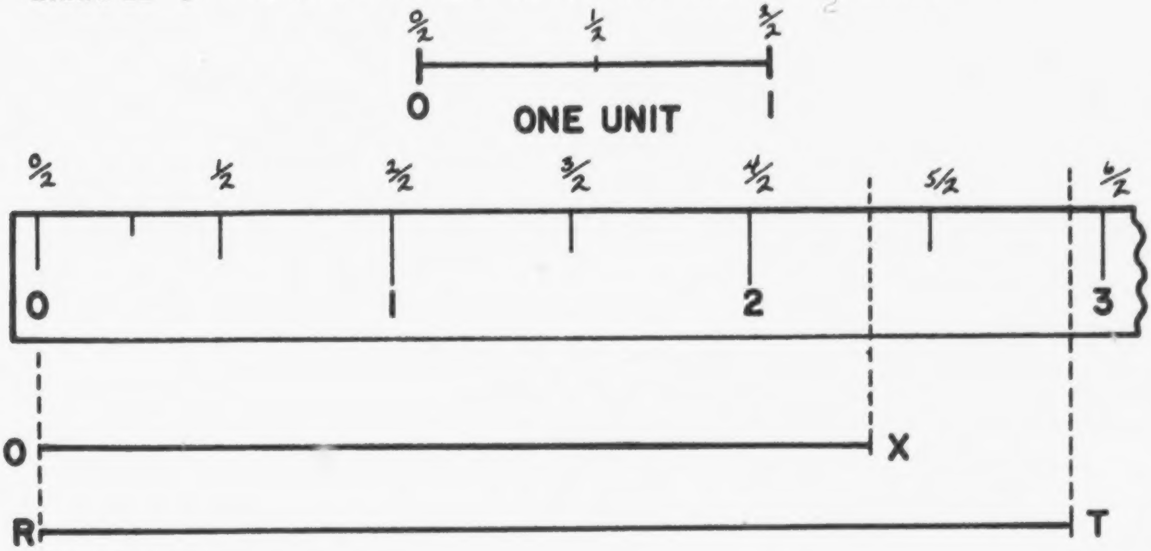
1) to the nearest unit.
2) to the nearest ½ unit.

Discussion questions which could be used are:

1) What difficulties arise when measuring to the nearest ½ unit? What could be done to help remedy these difficulties?

2) Describe how you would construct a "cubit ruler" ("hand ruler" or "foot ruler").

EXAMPLE 3: Measure OX and RT to the nearest $\frac{1}{2}$ unit.



SOLUTIONS :

For m($\overline{CX}$)	For m($\overline{RT}$)
$\frac{4}{2}$ units $< m(\overline{OX}) <$ $\frac{5}{2}$ units.	$\frac{5}{2}$ units $< m(\overline{RT}) <$ $\frac{6}{2}$ units.
m($\overline{OX}$) is closer to $\frac{5}{2}$ units.	m($\overline{RT}$) is closer to $\frac{6}{2}$ units.
To the nearest $\frac{1}{2}$ unit,...	To the nearest $\frac{1}{2}$ unit,...
$m(\overline{OX}) = \frac{5}{2}$ or $2\frac{1}{2}$ units.	$m(\overline{RT}) = \frac{6}{2}$ or 3 units.

CLASS ACTIVITY: PART 1

1. For each caveman unit of length pictured on page 8, draw a vertical line which you think divides the unit in half. The line for the cubit has been drawn as an example.

CONTENT AND APPROACH

The sample solutions on page 7 use $\frac{1}{2}$ unit as the unit of length. If needed, provide drill on and explanation of the process in changing improper fractions to mixed numbers and vice-versa. (See TR 1-5.) Some strategies are outlined below and on the following page.

Strategy I

$$\frac{7}{4} = \frac{4}{4} + \frac{3}{4} = 1 + \frac{3}{4} = 1\frac{3}{4}$$

$$1\frac{3}{4} = 1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4}$$

$$\frac{21}{8} = \frac{8}{8} + \frac{8}{8} + \frac{5}{8} = 1 + 1 + \frac{5}{8} = 2 + \frac{5}{8} = 2\frac{5}{8}$$

$$2\frac{5}{8} = 1 + 1 + \frac{5}{8} = \frac{8}{8} + \frac{8}{8} + \frac{5}{8} = \frac{21}{8}$$

Strategy II

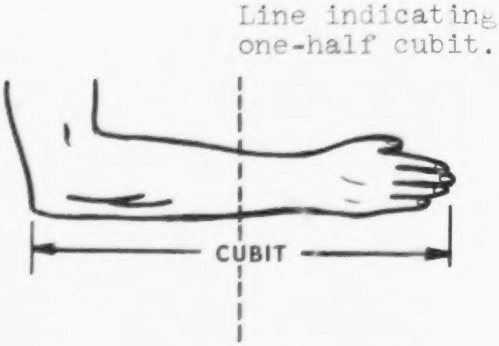
$$\frac{13}{4} = \ ? \qquad \begin{array}{r} 3 \\ 4\overline{)13} \\ \underline{12} \\ 1 \end{array} \qquad \frac{13}{4} = 3\frac{1}{4}$$

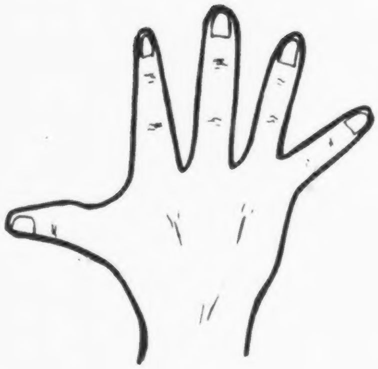
Divide the number of fourths, (13) by the number of fourths in one whole (4). The quotient (3) is the number of wholes, the remainder(1) is the number of fourths "left over".

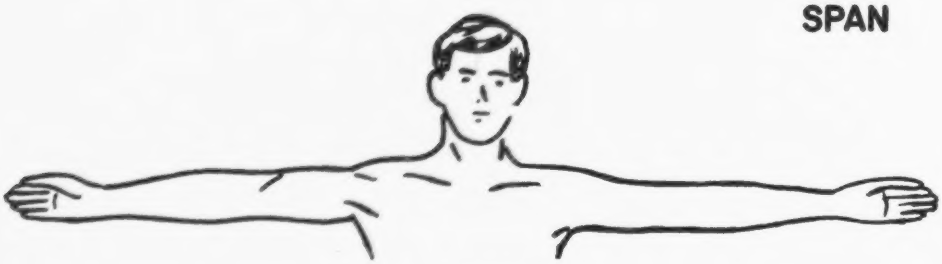
$$3\frac{1}{4} = \frac{(3 \times 4) + 1}{4} = \frac{12 + 1}{4} = \frac{13}{4}$$

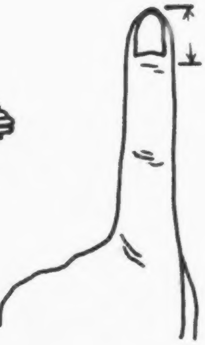
To obtain the number of fourths, multiply the number of wholes (3) by the number of fourths in one whole(4) and add the number of fourths indicated by the numerator of the fraction (1).

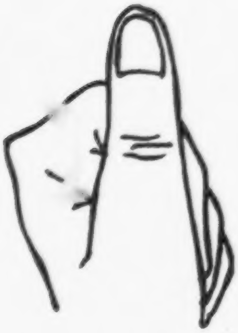
Use TR 1-6 prior to **CLASS ACTIVITY** (Part I). When using an arbitrary unit to measure to the nearest $\frac{1}{2}$ unit, a person must be able to locate the midpoint of the unit being used. After the students have made their guesses, the acetate overlay can be used to check the accuracy of the guesses.

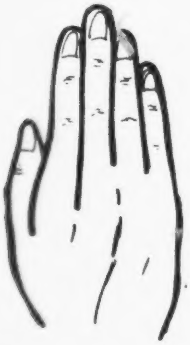

Line indicating one-half cubit.
CUBIT

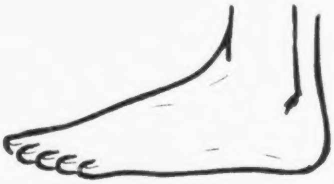

SPAN


FATHOM


FOREFINGER


THUMB


HAND


FOOT

2. Check your estimates. You will be given an acetate overlay for this purpose. How close were you?

CONTENT AND APPROACH

Each student should be responsible for making his or her own estimates for **CLASS ACTIVITY** (Part I). This is not designed to be used as a team activity.

Acetate overlays are provided so the students can check their estimates. Remember that there are no incorrect estimates — although some are better than others. You may wish to have an estimation contest to see who can make the best estimates.

CLASS ACTIVITY : Part II

1. (a) Select six distances in the classroom or vicinity, to be measured. (example: width of locker, length of room, height of desk,...)

   (b) Choose a unit of length from the list on page 4 and measure each distance to the nearest unit and $\frac{1}{2}$ unit. (This will require some estimating.)

   (c) Record your results in TABLE 1-1.

TABLE 1-1.

DISTANCE MEASURED	UNIT OF LENGTH USED	LENGTH TO	NEAREST..
		UNIT	$\frac{1}{2}$ UNIT

2. (a) Select one of the six distances from 1 (a).

   (b) Measure this distance to the nearest $\frac{1}{2}$ unit, using two units of length other than the one you used to get the measurement recorded in TABLE 1-1.

   (c) Record your results on the last two lines of TABLE 1-1.

DISCUSSION QUESTIONS

1. Compare your results in TABLE 1-1 with those of your classmates. Are they the same? Why or why not?

CONTENT AND APPROACH

The **CLASS ACTIVITY** (Part II) should be worked by teams of 3-4 students so that they can help each other "mark off" the distances and obtain the required measurements. Although the team members help each other, each person is responsible for recording only those measurements obtained by using his or her own caveman unit (span, hand,...).

Before the teams begin working, one distance should be measured, following the instructions 1(a) – 1(c) and illustrating how **TABLE I-I** is to be completed. Some students may wish to volunteer for such a demonstration. Emphasize the instructions for exercise 2.

Whether or not to give the teams some freedom in selecting the six distances is optional. To aid in the discussion of results, some distances should be selected for all students to measure. You may wish to provide a limited list (8 or 9) of distances to choose from or a combination of mandatory and student-selected distances to be measured.

The results of **TABLE I-I** will vary from person to person due to the variability of the caveman units.

THINGS TO DISCUSS

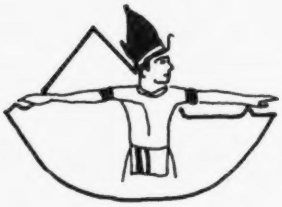
The **DISCUSSION QUESTIONS** (pages 9-10) outline topics for discussion related to Lesson 1 and in particular to the results of the **CLASS ACTIVITY** (Part 11).

Questions 1-3 in the **DISCUSSION QUESTIONS** demonstrate the variability of the caveman units. Of course, the choice of a unit is completely arbitrary. There is nothing incorrect about units being variable in length. The reason

Cont'd

2. Two persons measured the width of a desk to the nearest whole unit. One measured the width to be 6 hands, while the second person measured the same distance as 7 hands. Does this mean that one of the persons is incorrect? Why or why not?
3. If you need 30 cubits of rope and you send a friend to get it, will you probably get the length you wanted? Why or why not?
4. Suppose you measured the length of your classroom. Which unit of length, fathom or cubit, will give you the larger number for the length?
5. If one unit of length is longer than a second unit and you measure the same distance using both units, which unit gives the larger number? (See CLASS ACTIVITY; Part II, and question 4 on this page).
6. Describe the measurement involved when a football official "marks off" a 15 yard penalty.

WHERE DID IT COME FROM?



FATHOM: Dating at least as far as early Egyptian times, the fathom was the measure of a man's outstretched arms: 5, $5\frac{1}{2}$, and now 6 feet in length.

for uniform units of length is to prevent chaos in communication with others, not because it is inherently wrong to use units of variable length.

For Question 4, one advantage would be their availability. The variability of length is probably the major disadvantage.

Question 5 illustrates the effect of the unit on the measure. A discussion of what units are appropriate to measure a given distance would be relevant to this question.

Question 6 illustrates a common situation in which a distance is "marked off" using a unit related to the caveman units. A related question for the football fans in your class would be "What is used to measure the 10 yard distances which the offense needs to gain in 4 downs to keep possession of the ball?". Which is more accurate, the "marking off" of penalties or the measuring for first down yardage?

A possible discussion question related to the fathom is "What possible reasons could be given for the fathom increasing in length?" (On the average, men have grown larger over the years).

As a class activity, the class could write an article describing various distances and dimensions of the school (width of corridor, locker...) in caveman units for publication in a school newspaper or placement on bulletin boards.

A display of linear measuring instruments could also be collected and set up in the room, asking the students to bring in items for the display.

LET'S COMPARE

Many daily uses of measurement require only an estimate. Using parts of the body as units of measure (span, cubit,...) will never replace the foot, inch, meter, and centimeter. However, if caught without a ruler or if only an accurate estimate is required, the caveman units used in Lesson 1 should be precise enough to handle many household and shop situations.

One approach to obtaining an accurate estimate to the width of your text is described below.

EXAMPLE: Estimate the width of your text.

SOLUTION :

1. Select a unit of length. Suppose we use the thumb (which is about 1 inch wide).

2. Use this unit (thumb width) to "step off" the desired length.

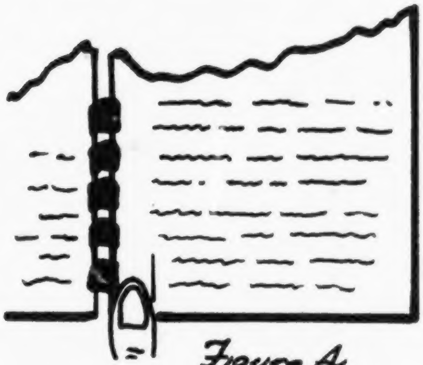


Figure A

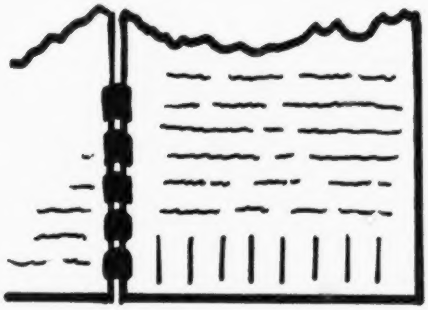


Figure B

3. In this example, the text is about 9 thumbs wide. Since a thumb is about 1 inch wide, your estimate would be 9 inches.

LET'S COMPARE

OBJECTIVES

1. Each student shall determine the length of his own cubit, span, hand,... to the nearest ½ inch, obtaining equivalents in feet and inches for his caveman units.

2. Given a measurement in caveman units, the student shall be able to give an equivalent length in feet and inches. (Example: 2 hands=? inches)

EQUIPMENT AND TEACHING AIDS

A. STUDENT

1. Masking tape.
2. 100 ft. or 50 ft. tape measures. {For Standardizing Pace
*3. Ruler (½ inch with equal length markings).

B. TEACHER

1. Overhead projector.
2. Projection screen.
*3. Prepared transparencies.
*4. Acetate ruler (½ inch with equal length markings).

CONTENT AND APPROACH

The student is not asked to work problems such as the one illustrated on page 11 until Lesson 3. This example is included to indicate the usefulness of caveman units as guides for estimating length. Lesson 2 enables the student to obtain equivalents in feet and inches for his own caveman units so that he can use them as estimation guides. (Lesson 1 concentrated on measuring distances in caveman units.)

DISCUSSION QUESTIONS

1. In the example on page 11, what other caveman units could have been used instead of the thumb to obtain an accurate estimate for the width of the text?

2. Would the cubit have been a suitable unit of length to use in the example on page 11? Why or why not?

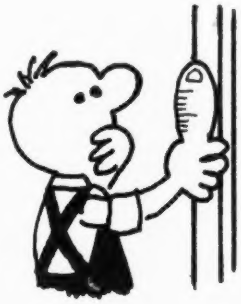
DEVELOP YOUR OWN METHODS . . .

Being able to estimate accurately is an important skill with many useful applications. You should develop your own methods of estimating distances. There is no one right way to estimate.

Lessons 2 and 3 use caveman units as guides for estimating distances. Other estimation guides could be used but caveman units are among the most convenient ones to use.

RULE OF THUMB. . .

This expression means a reliable rule for rough calculations (or estimations). It refers to the fact that people once used the first joint of the thumb as a one-inch measure.



THINGS TO DISCUSS

The **DISCUSSION QUESTIONS** stress the need for a unit of length not only to be a length, but also "suitable" for the distance under consideration. Selecting a unit "suitable" for a given distance will be discussed in more detail in Lesson 12, pp. 76-80.

CONTENT AND APPROACH

The statements under **DEVELOP YOUR OWN METHODS** are essential to the spirit in which Lessons 1-3 are taught. Do not limit the student to the methods illustrated in the text. *There is no one right way to estimate*, and the student should be encouraged to develop his own methods of estimation. Caveman units are used to estimate lengths because of their convenience (we always carry them with us) and their significance in the historical development of linear measure. Items for discussion at a later date (questions 2-4, p.16) are other lengths that could be used as estimation guides and the properties of a "good" estimation guide.

CLASS ACTIVITY I

With the aid of a partner, obtain these measurements for yourself to the nearest $\frac{1}{2}$ inch. Record your results in TABLE 2-1.

TABLE 2-1

DESCRIPTION OF UNIT	LENGTH TO NEAREST $\frac{1}{2}$ INCH
a) Width of THUMB	
b) Distance from first joint to end of FOREFINGER	
c) HAND	
d) SPAN	
e) CUBIT	
f) Length of FOOT	
g) Distance from nose to thumb of outstretched arms (ARM)	
h) Distance across outstretched arms (FATHOM)	
i) PACE	

COMPARING SIZES . . .

Compare your measurements in TABLE 2-1 with those of your classmates. Are they the same? Why or why not?

Complete TABLE 2-2. Use the longest and shortest lengths in your class for the units listed.

TABLE 2-2

UNIT	LONGEST LENGTH	SHORTEST LENGTH	DIFFERENCE
Width of thumb			
Hand			
Span			
Cubit			
Pace			

CONTENT AND APPROACH

To obtain foot and inch equivalents for his caveman units (**TABLE 2-1**), the student must measure to the nearest $\frac{1}{2}$ inch. If the review is needed, the overhead transparencies may be used to review the meaning of "measuring to the nearest $\frac{1}{2}$ inch".

The rulers on the transparencies are marked like the $\frac{1}{2}$ inch rulers provided for the students (although different units of length are used).

Ask the student to give the measure of each segment...

1) to the nearest unit.
2) to the nearest $\frac{1}{2}$ unit.

A 100 foot or 50 foot tape measure is useful for standardizing the pace and measuring some of the longer caveman units. Most tape measures are graduated to the nearest $\frac{1}{4}$ inch. You may need to indicate which markings indicate halves before gathering the data for **TABLE 2-1** .

To obtain the data for **TABLE 2-1** , each student will need the aid of a partner. You may wish to divide the class into teams of 3-4 students each.

Due to the variability of one's own pace when walking, it is difficult to get an accurate measurement of the pace by measuring the distance covered by only one pace. One method which could be used is described below.

> To standardize your pace, walk a convenient number of paces (10, 20, 30,...) and measure this distance (to the nearest inch). To get the average length of one pace, divide by the number of paces.

The problem of accurately measuring the pace could be brought out as a topic for discussion, allowing students an opportunity to develop a procedure similar to the one above on their own.

TABLE 2-2 should be done with the entire class after the teams complete **TABLE 2-1** . The data in **TABLE 2-2** will demonstrate the variability of caveman units.

EXERCISES

1. Using your own measurements, (see TABLE 2-1) complete the following relationships between the "caveman units" and our present units of length. (≈ is read "approximately the same as.")

   a. 1 thumb ≈ ______ in.
   b. 1 span ≈ ______ in.
   c. 1 hand ≈ ______ in.
   d. 1 cubit ≈ ______ in. ≈ ______ ft.
   e. 1 pace ≈ ______ in. ≈ ______ ft.
   f. 1 fathom ≈ ______ ft.
   g. 1 arm ≈ ________ ft.

2. Given the following lengths, convert them to feet and inches.

   a. 7 forefingers ≈ ______ in.
   b. $2\frac{1}{2}$ spans ≈ _____ in.
   c. 4 hands ≈ ______ in.
   d. 2 cubits ≈ ____ ft.
   e. 10 paces ≈ _____ ft.
   f. $2\frac{1}{2}$ hands ≈ _____ in.
   g. $3\frac{1}{2}$ thumbs ≈ ____ in.
   h. $\frac{1}{2}$ cubit ≈ _____ in.

STOP

CHECK YOUR RESULTS FOR EXERCISES 1 AND 2
BEFORE GOING TO THE DISCUSSION QUESTIONS

DISCUSSION QUESTIONS

1. Which of the units in TABLE 2-1 could you use as an approximation for an inch? a foot? a yard?

2. Will the answers to question 1 be the same for all students in the class? Why or why not?

3. Which of the units in TABLE 2-1 would be convenient for you to use as a guide for estimating distance? Why?

4. Will the answer to question 3 be the same for all students in the class? Why or why not?

ANSWERS

EXERCISES

Use the following results as a guide. Results may vary among members of the class.

1. a. 1 thumb ≈ 1 in.
   b. 1 span ≈ 8 in.
   c. 1 hand ≈ 4 in.
   d. 1 cubit ≈ 19 in. ≈ $1\frac{1}{2}$ ft.
   e. 1 pace ≈ 60 in. ≈ 5 ft.
   f. 1 fathom ≈ 6 ft.
   g. 1 arm ≈ 3 ft.

2. a. 7 forefingers ≈ 7 in.
   b. $2\frac{1}{2}$ spans ≈ 20 in.
   c. 4 hands ≈ 16 in.
   d. 2 cubits ≈ 3 ft.
   e. 10 paces ≈ 50 ft.
   f. $2\frac{1}{2}$ hands ≈ 10 in.
   g. $3\frac{1}{2}$ thumbs ≈ $3\frac{1}{2}$ in.
   h. $\frac{1}{2}$ cubit ≈ $9\frac{1}{2}$ in.

The answers to exercises 1 and 2 of the **EXERCISES** , are meant to be approximations to the $\frac{1}{2}$ unit. Which would be useful as a guide for estimation? Some of the data in exercise 1 could be used to discuss the properties of a good estimation guide (easy to remember, easy to compute with,...). For example, if the result for #1d was 1 cubit ≈ $17\frac{1}{2}$ inches ≈ $1\frac{1}{2}$ feet, which would be most convenient to use? (In most cases, $1\frac{1}{2}$ feet would be the most convenient equivalent.) This discussion could be delayed until discussion question 4 (Lesson 3, p. 16)concerning properties of a good estimate.

The given data in exercise 2 could be interpreted as information gathered to obtain an estimate. For example, if the distance is about 7 forefingers long, an estimate for its length would be how many inches?

DISCUSSION QUESTIONS

Differing human measurements will cause a variety of answers. In general, the convenience of a particular unit for estimating will be related to its ease in computation.

EDUCATED GUESSING

CLASS ACTIVITY 1

Follow the instructions below. Record all results in TABLE 3-1.

1. Select six distances to be measured. List the distances under DISTANCE MEASURED.

2. Measure each distance to the nearest $\frac{1}{2}$ unit with some caveman unit and record your results under MEASURED LENGTH (caveman units).

3. Convert the measurements in# 2 to their equivalents in feet and inches. Record these results under ESTIMATE (feet and inches).

4. Check your estimates by using a ruler, tape measure, or yardstick to measure each of these six distances to the nearest $\frac{1}{2}$ inch. Record these measurements under MEASURED LENGTH (nearest $\frac{1}{2}$ inch).

5. See how close your estimate is to the measured distance by finding the difference between the measured and estimated distances. Record the differences under DIFFERENCE.

TABLE 3-1

DISTANCE MEASURED	MEASURED LENGTH (caveman units)	ESTIMATE (feet and inches)	MEASURED LENGTH (nearest $\frac{1}{2}$ inch)	DIFFERENCE

EDUCATED GUESSING

OBJECTIVES

1. Given a distance from 1" to 40', the student shall use caveman units to estimate its length. Estimates will be checked by computing the difference between the measured distance and the estimate.

2. Given a description of a distance such as ...

   3 paces < Length A < 4 paces.

   To the nearest whole pace, m(Length A) = 4 paces

   ... the student shall give an estimate of a distance with these properties by indicating the endpoints of their estimate with two pieces of tape on a flat surface. Estimates will be checked by measuring with the given caveman unit.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

*1. Rulers graduated in ½ inches.

2. 100 ft. or 50 ft. tape measure (2 or 3 per class).

3. Yardstick (1 for each 3-4 students).

4. Masking tape.

B. TEACHER

1. Postal scale
2. 25-30 pennies
3. 1 quarter
4. 1 dollar bill

CLASS ACTIVITY I can be worked by teams of 3-4 students. Students should practice making conjectures and estimates. Therefore, do not consider any estimates as incorrect (although some come close). An indication of the accuracy of the estimates can be given by comparing the differences between the estimate and measured length. **ANSWERS** for **TABLE 3-1** will vary.

If all students are given the same distances to measure for **TABLE 3-1**, a team or individual competition could be used to see who is the best estimator.

DISCUSSION QUESTIONS

1. What disadvantages would occur if parts of the body were used as units of length and as measuring instruments instead of our present units and rulers? What advantages?

2. The following statement was made on page 12. "Other estimation guides could be used but caveman units are among the most convenient ones to use." Do you agree with this statement? Why or why not?

3. What are some other lengths which could be used as guides for estimating distances?

4. To be useful, what properties must a "good" estimation guide have?

CLASS ACTIVITY II

1. Estimate by sight (without "marking off") distances with the following properties. Indicate the endpoints of each estimate by placing two pieces of tape that distance apart on a flat surface (desk, floor,...).

2. Check each of your guesses by measuring with the given caveman unit.

a. 3 cubits < Length A < 4 cubits
to the nearest whole cubit,
m(Length A) = 3 cubits.

b. 6 paces < Length B < 7 paces
to the nearest whole pace,
m(Length B) = 7 paces.

CONTENT AND APPROACH

[illegible] DISCUSSION QUESTIONS [illegible] CLASS ACTIVITY I [illegible]

DISCUSSION QUESTIONS

[illegible]

[illegible] Several possibilities exist—[illegible]

CLASS ACTIVITY II

This estimation activity is designed as a team activity ([illegible]-4 students per team). Students will check their own estimates by measuring.

c. 2 spans $<$ Length C $<$ 3 spans.
To the nearest whole span,
m(Length C)=3 spans.

d. 6 thumbs $<$ Length D $<$ 7 thumbs.
To the nearest whole thumb,
m(Length D) = 6 thumbs.

DO YOU AGREE ?

One writer has stated that "with a little practice you should be able to measure from a half-inch to 8 inches with just your hand--and come to within a sixteenth of an inch every time."

Mechanix Illustrated, September 1963, p.26.

MORE EDUCATED GUESSING ...

Your answers to the following questions should be educated guesses. Do not do any computation. For each multiple-choice question, you are to select the answer that you believe to be the best estimate.

1. What is the length in inches of the diameter of a quarter?

a) $\frac{3}{4}$ in. b) 1 in. c) $1\frac{1}{4}$ in. d) $\frac{1}{2}$ in. e) $1\frac{1}{2}$ in.

2. The length of a dollar bill is about:

a) 6 in. b) 7 in. c) 8 in. d) 9 in. e) 10 in.

MORE EDUCATED GUESSING contains a variety of estimation activities. Check the results by measurement, after each student has made his guess. A discussion of the results could focus on the various approaches which were used to obtain the estimates in question. In the last 4 questions, discuss the relationship between questions 1 and 2 and the relationship between questions 3 and 4. This section is optional.

ANSWERS

1. 1 inch
2. 6 inches

3. The length of a full-sized automobile is about:

   a) 12 ft. b) 18 ft. c) 25 ft. d) 10 ft. e) 30 ft.

4. The length of this text book is about:

   a) 6 $\frac{1}{2}$ in. b) 8 $\frac{1}{2}$ in. c) 11 in. d) 15 in. e) 1 ft.

After completing your educated guessing, try to find answers to the above questions. Careful measurement will enable you to determine the best estimate for each situation.

Now try these.

1. About how many pennies in an ounce?
2. Approximately how many pennies in a pound of pennies?

   a) 60 b) 120 c) 180 d) 240 e) 300
3. About how many pennies (stacked on top of each other) to make a stack one inch high? one foot high?
4. About how many pennies would have to be stacked one on top of the other to reach from the floor to the ceiling of a room eight feet high?

   a) 600 b) 800 c) 1100 d) 1600 e) 2200

KEEP IN PRACTICE . . .

The ability to make reasonable estimates is not a skill which can be developed in a couple of lessons. Keep in practice by estimating whenever possible. If you can make reasonable estimates, you will be able to check the reasonableness of any measuring you do.

ANSWERS

[illegible]. 1[illegible] feet
[illegible]. [illegible] inches

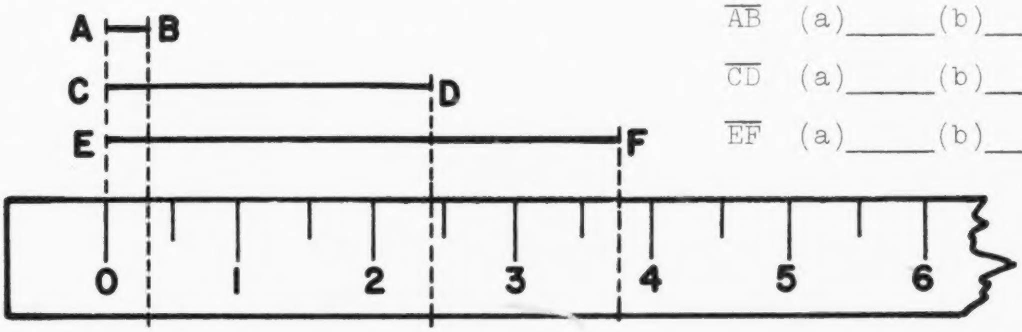
1. 1[illegible] pennies
[illegible]. 1[illegible] (using 1[illegible] per inch)
[illegible]. 16; [illegible] (using 16 per inch)
[illegible]. 1[illegible] (Using 1[illegible] per inch, the answer is [illegible].)

The statements under KEEP IN PRACTICE indicate the spirit in which much of the material in this booklet is presented. Many of the EXERCISES contain problems in which the student is required to estimate lengths. If a student is expected to develop an ability to make reasonable estimates, a brief or one-time exposure is not enough. Ask your students to estimate whenever appropriate.

✓ POINT

Use the following questions and problems to check your understanding of Lessons 1-3. If you have difficulty with any question, it may indicate a topic you should review.

1. Use the ruler shown to measure each segment to the nearest a) unit, and b) $\frac{1}{2}$ unit.



$\overline{AB}$ (a)_____(b)_____

$\overline{CD}$ (a)_____(b)_____

$\overline{EF}$ (a)_____(b)_____

2. For each measurement in caveman units, give an equivalent length in feet and inches.

a. 2 spans ______ c. $1\frac{1}{2}$ hands ______ e. 2 cubits ______
b. 3 paces ______ d. 5 thumbs ______ f. 3 arms ______

3. Arrange the following units in order of length from smallest to largest. (Indicate the smallest by 1, next smaller by 2,...)

_______ hand
_______ fathom
_______ forefinger
_______ foot
_______ span
_______ pace
_______ arm
_______ cubit

4. You should be able to....

   a. demonstrate how caveman units can be used to estimate distances.

   b. list some advantages and disadvantages of using caveman units to measure length.

ANSWERS

PRECISION

1.

Segment	Unit	$\frac{1}{2}$ Unit
$\overline{AB}$	0	$\frac{1}{2}$
$\overline{CD}$	2	$2\frac{1}{2}$
$\overline{EF}$	4	4

*2. a. 16 inches

b. 15 feet

c. 6 inches

d. 5 inches

e. 3 feet

f. 9 feet

3. 2 hand

8 fathom

1 forefinger

4 foot

3 span

7 pace

6 arm

5 cubit

4. No response is intended. These statements are to be used as a self-check by the student.

*Use these answers as a guide. Results may vary among members of the class.

ESTIMATES ARE NOT ENOUGH

BECOMING MORE PRECISE...

Estimation is useful for many everyday household and industrial situations. However, industry also needs measurements that are extremely accurate and precise. Industrial uses often demand measurements to the nearest thousandth and even millionth of an inch.

All measurements are approximations. The science of measurement is the art of obtaining better approximations. A more precise measure of a length is one in which a better approximation of the length is obtained.

EXAMPLE

$\overline{OX}$ has been measured using two different units of measure. Which unit gives the more precise measure (the better approximation of the length of $\overline{OX}$)?

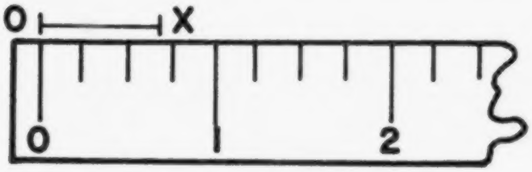


Figure 1

When measured to the nearest $\frac{1}{4}$ inch,...

$\frac{2}{4}$ inches $< m(\overline{OX}) <$ $\frac{3}{4}$ inches.

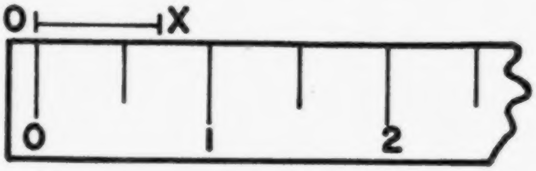


Figure 2

When measured to the nearest $\frac{1}{2}$ inch,...

$\frac{1}{2}$ inch $< m(\overline{OX}) <$ $\frac{2}{2}$ inches.

ESTIMATES ARE NOT ENOUGH

OBJECTIVES

1. To indicate the need for precision and uniform standard units in an industrial society.

2. To introduce the following terminology:
   a) precision b) standard unit

3. Given two measurements, the student shall be able to indicate which is more precise.

EQUIPMENT AND TEACHING AIDS

STUDENT

1. Dictionary and encyclopedia (for page 28).

CONTENT AND APPROACH

Lesson 4 is a transitional lesson between the estimation activities and use of caveman units in Lessons 1-3 and more precise measurement (nearest .1 cm., $\frac{1}{16}$ in.) and use of present-day standard units (English and metric) in the second part of this booklet.

The discussion on pages 20 and 22 indicates the reliance of industry upon precision measurement and gives a definition of precision. The more precise measurement is the one which gives the better approximation of the length.

The second paragraph is related to the statement of page 2. By measuring more precisely we are obtaining better approximations, reducing the size of the error between the actual length and the number used to express the length.

SOLUTION

In Figure 1:

$m(\overline{OX})$ lies somewhere in the interval between $\frac{2}{4}$ inches and $\frac{3}{4}$ inches. The measure is located in an interval equal in length to $\frac{1}{4}$ inch, smaller the unit of measure.

In Figure 2:

$m(\overline{OX})$ lies somewhere in the interval between $\frac{1}{2}$ inch and $\frac{2}{2}$ inches. The measure is located in an interval equal in length to $\frac{1}{2}$ inch, the unit of measure.

Which of the two measurements is more precise? The more precise measurement will give the better approximation of the length of the segment.

Since the interval is shorter in Figure 1, the measure obtained for $\overline{OX}$ in Figure 1 is a better approximation of its actual length.

Therefore, measuring to the nearest $\frac{1}{4}$ inch gives a more precise measure than measuring to the nearest $\frac{1}{2}$ inch because it locates $m(\overline{OX})$ in a shorter interval.

PRECISION — depends on the unit of measure. The smaller the unit of measure, the more precise the measurement. Thus, a measurement made to the nearest $\frac{1}{16}$ inch is more precise than one made to the nearest $\frac{1}{10}$ inch.

CONTENT AND APPROACH

The question asked on page 20 is the type which the student should be able to answer, indicating his understanding of the definition of precision.

The sample solution on page 21 uses the definition of precision given on page 20. When a smaller unit is used, the length is located in a shorter interval. As a result, a better approximation of the length is obtained.

Notice that the summarizing definition at the bottom of one page emphasizes the reliance of precision upon the unit of measure.

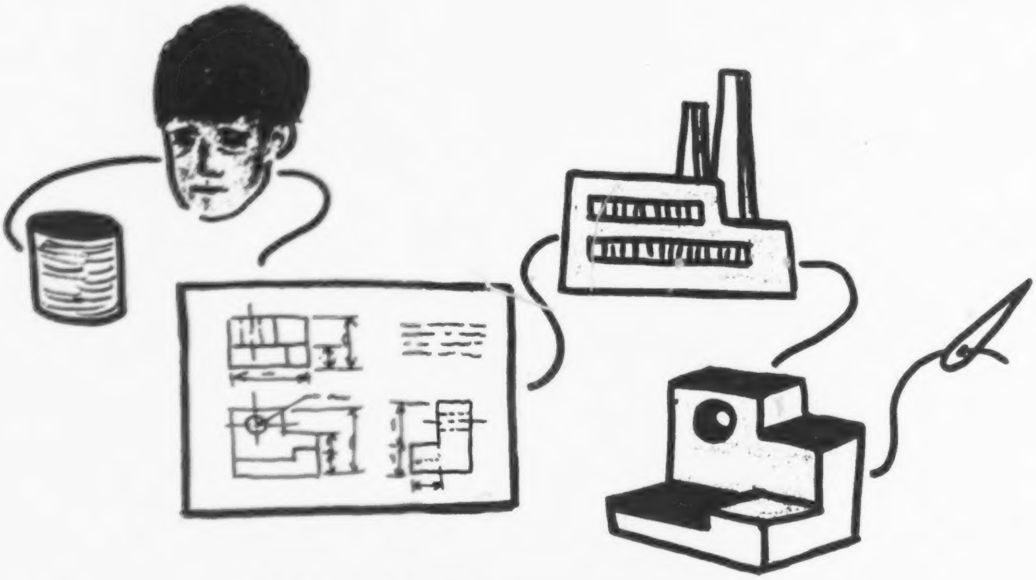
If a follow-up activity to a discussion of the sample solution is needed to aid the students' understanding of precision, use exercises similar to # 2(d) on page 25 and # 4 on page 26. The sample solution can be used as a model for thinking through the question "Which is more precise?".

DISCUSSION QUESTIONS

1. What is $m(\overline{OX})$ when measured to the nearest $\frac{1}{4}$ inch? to the nearest $\frac{1}{2}$ inch?
2. Explain how using a smaller unit of measure gives a better approximation for the length of a segment.

PRECISE MEASUREMENTS ARE NEEDED

Mass production demands that everything fit together just right. The assembly line emerged as man learned to measure more accurately and precisely. The blueprint is the only thread between the designer and the men who make the product. The assembly line cannot operate unless the blueprint dimensions can be accurately reproduced. Precise measuring instruments and methods of measuring make this possible.



THE BLUEPRINT IS THE ONLY THREAD BETWEEN THE DESIGNER AND THE MEN WHO MAKE THE PRODUCT.

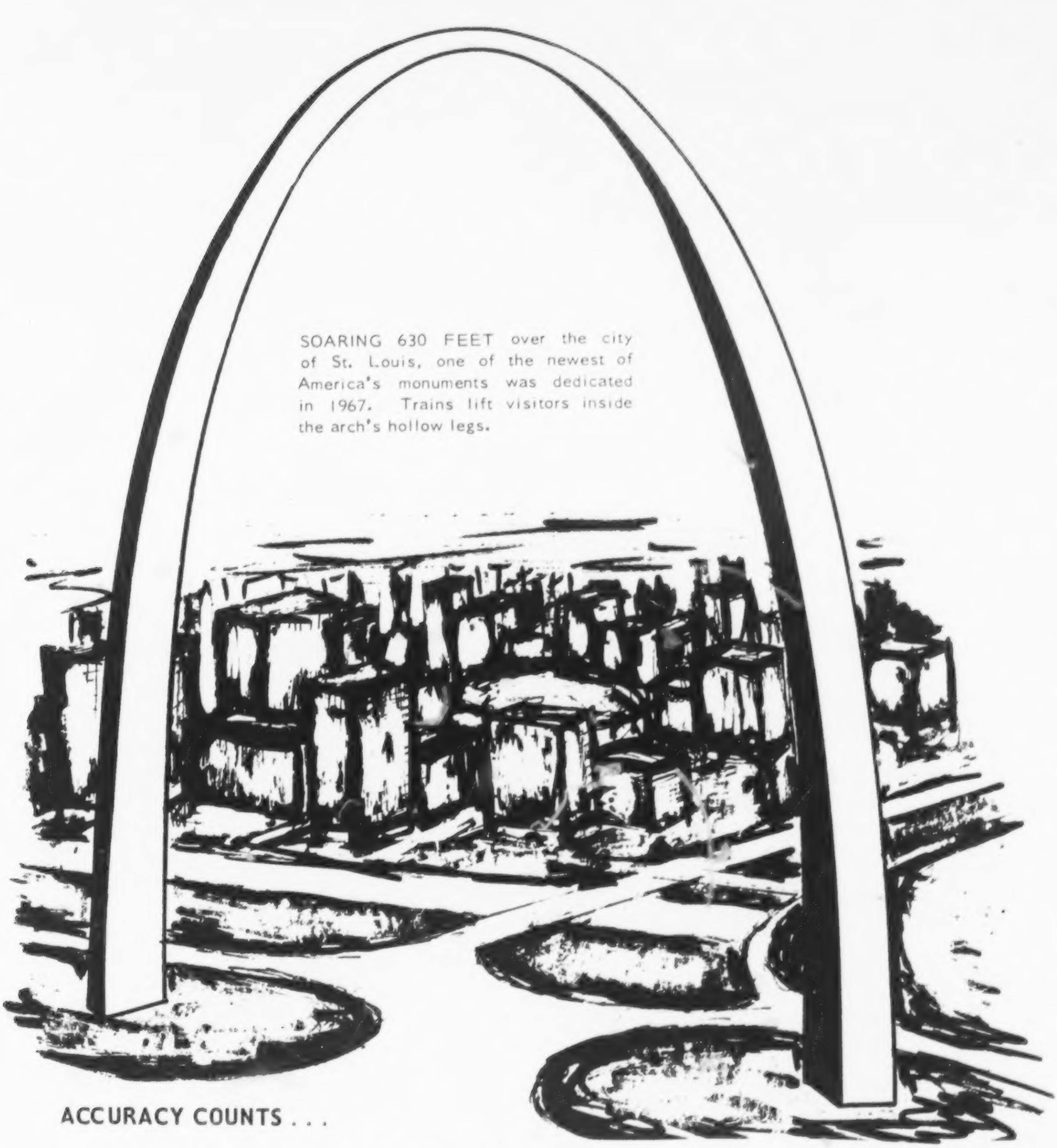
DISCUSSION QUESTIONS

1. This question is not directly related to the discussion of precision and is inserted as a review of measuring skills developed in the booklet. $m(\overline{CX}) = \frac{3}{4}''$ (nearest $\frac{1}{4}''$); $\frac{1}{2}''$ (nearest $\frac{1}{2}''$).

2. The smaller unit will locate the length in a shorter interval, giving a better approximation of the length.

CONTENT AND APPROACH

The bottom half of page 22 and the picture on page 23 give further emphasis to the need for precision measurements in industry. The discussion and development of a definition for precision will be relatively meaningless to the student unless he is given at least some indication of the need for precision.



SOARING 630 FEET over the city of St. Louis, one of the newest of America's monuments was dedicated in 1967. Trains lift visitors inside the arch's hollow legs.

ACCURACY COUNTS . . .

When constructing the 630-foot Gateway Arch in St. Louis, an error of $\frac{1}{64}$ inch in pouring the foundations would have meant that the legs of the arch could not meet. Fortunately, this error was avoided!

✓ POINT

Circle the more precise measurement in each pair.

To the nearest... a. .1 m. .01 m.

b. $\frac{1}{10}$ in. $\frac{1}{8}$ in.

c. ft. in.

d. $3\frac{3}{4}$ in. to the nearest $\frac{1}{4}$ in.

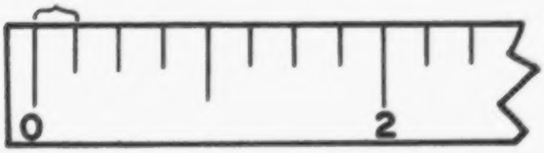
$6\frac{7}{8}$ in. to the nearest $\frac{1}{8}$ in.

e. 3.02 cm. to the nearest .01 cm.
1.4 cm. to the nearest .1 cm.

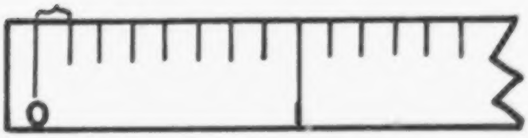
EXERCISES

The following exercises begin a review and summary of some basic skills that enable a person to master the reading of a ruler to varying degrees of precision.

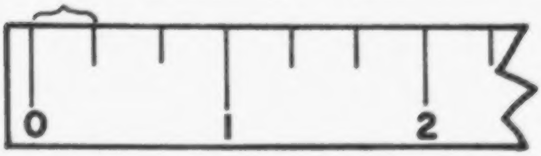
1. Answer the following questions for each of the rulers pictured.
   a. How many sub-divisions per unit?
   b. The size of the smallest sub-division on the ruler is what fractional part of a unit?



I. a.________ b.________

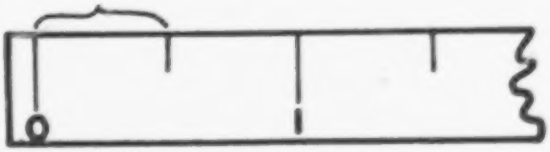


II. a.________ b.________



III. a.________ b.________

A brace (⏞) is used to indicate one sub-division on each ruler.



IV. a.________ b.________

ANSWERS

✓ POINT

To the nearest,......

a. .01 m

b. $\frac{1}{10}$ in.

c. in.

d. $6\frac{7}{8}$ in. to the nearest $\frac{1}{8}$ in.

e. 3.02 cm. to the nearest .01 cm.

The above are the more precise in each pair and should have been circled.

EXERCISES

I. a. 4 b. $\frac{1}{4}$

II. a. 8 b. $\frac{1}{8}$

III. a. 3 b. $\frac{1}{3}$

IV. a. 2 b. $\frac{1}{2}$

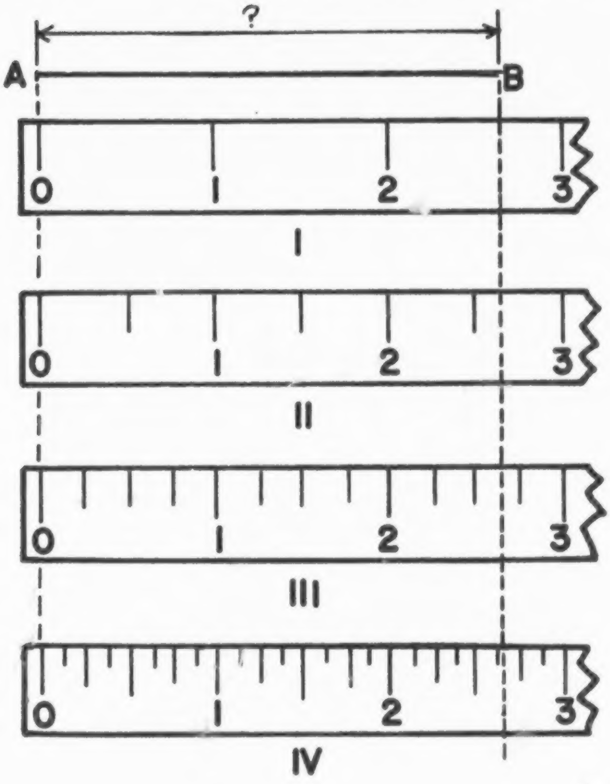
. Each of the measurements below is to a different degree of precision.

a. Which measurement(s) is (are) correct? ________

b. Which measurement is "best"? ________

c. Which measurement(s) is (are) "exact"? ________

d. Which measurement is the most precise? ________



$m(\overline{AB})$ is read "the measure of segment AB."

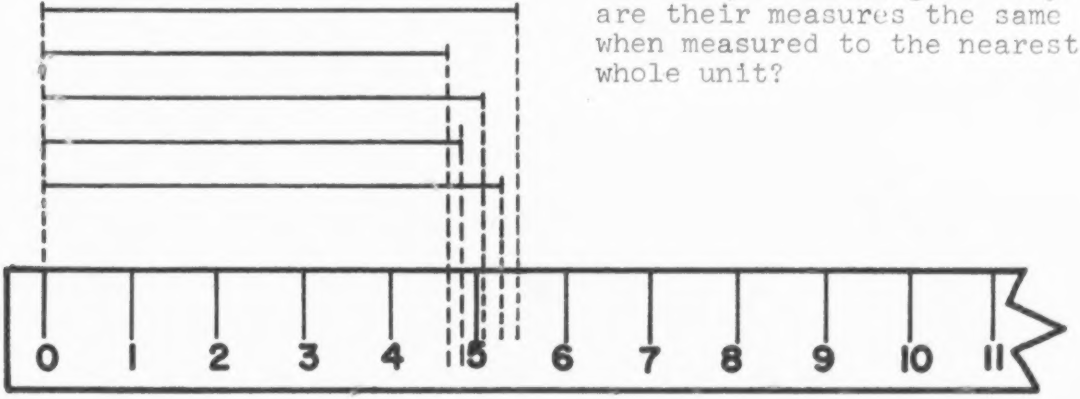
$m(\overline{AB}) = 3$ units to the nearest unit.

$m(\overline{AB}) = \frac{5}{2}$ or $2\frac{1}{2}$ units to the nearest $\frac{1}{2}$ unit.

$m(\overline{AB}) = 1\frac{1}{4}$ or $2\frac{3}{4}$ units to the nearest $\frac{1}{4}$ unit.

$m(\overline{AB}) = \frac{21}{8}$ or $2\frac{5}{8}$ units to the nearest $\frac{1}{8}$ unit.

3. The segments to the left are unequal in length. Why are their measures the same when measured to the nearest whole unit?



ANSWERS

2. a. All are correct.
   b. Depends on the purpose of the measurement.
      If the students answer $2\frac{5}{8}$ in. (IV), discuss

      why this may not necessarily be the best or
      most appropriate measurement.
   c. None are exact.
   d. The measurement to the nearest $\frac{1}{8}$ in. is most
      precise.

3. Although of unequal length, each segment is closer in length to 5 units than any other whole unit marking. This exercise indicates the distinction between the length and the measure of a segment. This is an important idea but should not be belabored at this stage.

NOTE: When we say the length of some distance is the same as the length of another, we mean that we are unable to detect a difference in their lengths. However, with magnification or more careful measuring, a difference might be detected because all measurements are only approximations.

4. For each pair circle the phrase that indicates the more precise measurement.

To the nearest...	a.	$\frac{1}{2}$ inch	$\frac{1}{4}$ inch
	b.	.1 cm.	.01 cm.
	c.	$\frac{1}{16}$ cubit	$\frac{1}{8}$ cubit
	d.	$\frac{1}{8}$ inch	$\frac{1}{2}$ inch
	c.	$\frac{1}{2}$ inch	$\frac{1}{2}$ foot

EVERY MAN FOR HIMSELF . . .

One disadvantage to using parts of the body as units of length is that the size of each unit differs from person to person. This did not create serious problems for the caveman. Each family was independent. They did not rely on others to provide food, clothing, shelter and other daily needs. As a result, each locality used units that fit their own needs without much consideration for the world around them. Therefore, units of measure varied from town to town and sometimes from family to family.

STANDARD UNIT —— a unit of measure agreed upon and used by a large number of people.

✓ POINT

1. Name some standard units of length that are currently used.

ANSWERS

[illegible] in.

[illegible] in.

[illegible] units

[illegible] in.

[illegible] in.

[illegible]

CONTENT AND APPROACH

The need for uniform standard units is [illegible] in the discussion on pages [illegible] and [illegible]. It is important for the student to understand that there is nothing inherently incorrect about units that vary in length, [illegible] of these as guides for estimation. However, [illegible] standards become essential for communication purposes when we begin to rely on other persons to produce products to our specifications.

Page [illegible] summarizes the need for standard units in societies whose members are dependent on each other for goods and services.

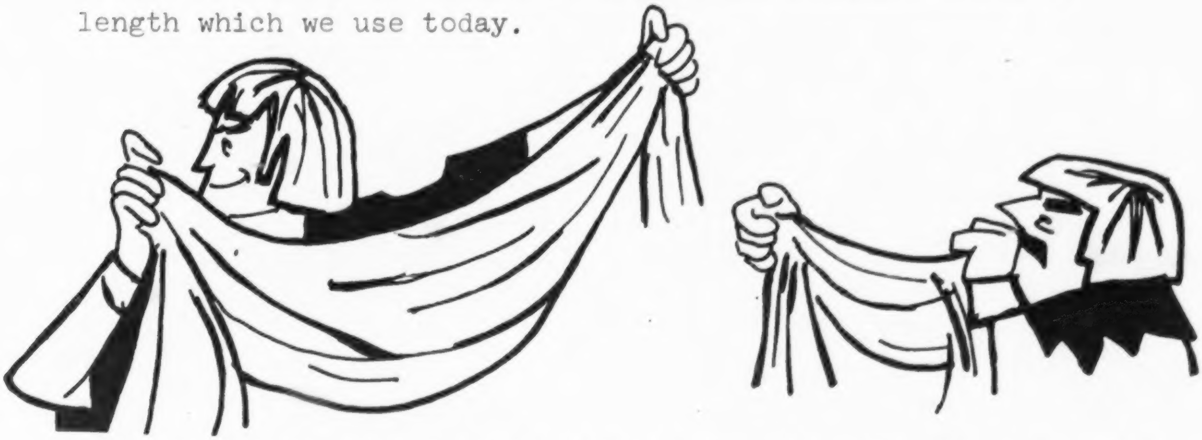
EVERY MAN FOR HIMSELF indicates the limited need for standard units in a society like that of the cavemen. New methods and procedures are developed as the need arises.

✓ POINT

Student response to this question may give some indication of their familiarity with the units formally introduced (especially the metric units) in Lesson [illegible].

NO MAN IS AN ISLAND . . .

The units which we use for measuring have interesting histories. Mankind's need for standard units of length has evolved over centuries in a way quite similar to the development outlined in this booklet: The early inch was the width of a man's thumb, or that of three barley corns; the foot was the length of a man's foot; and the yard was the distance from the nose to the fingertips of an outstretched arm. Since these units of length varied from one person to another, it became necessary to establish the common standard units of length which we use today.



As man developed an industrial society, the need for units that were the same regardless of locality became more apparent. Universally accepted standard units became mandatory with the introduction of assembly-line techniques in the early 1800's.

Some units became widely used and were standardized by law or agreement and used by entire countries or regions.

The first known standard units of measure were established about 8000 years ago. Our present standards originated much later.

1790 - metric system developed in France
1866 - use of metric system legalized in U.S.
1959 - present United States inch standardized

CONTENT AND APPROACH

The first paragraph extends the discussion on page 26 concerning the need for and development of standard units of measure.

The second half of the page indicates the recent development of our present standards. **HOW LONG IS AN INCH?** (p. 28) gives some background on the 1959 standardization of the present United States inch.

The necessity for standard units of measure can be illustrated by a situation such as the following. Suppose you were asked to measure the length of a chalk board with any object you have available. Results such as the following could be obtained.

Unit of Measure	Measurement
pencil	15 pencils
stick	4 sticks
paper clip	103 paper clips
books	15 books

Can the length of the chalkboard be visualized from any of these measurements? You probably have a general awareness of the length of one paper clip, but few persons can picture with any accuracy a length of 103 paper clips laid end to end. Books are of many sizes, making " 15 books long" rather meaningless. Similar remarks can be made for each of the measurements. Therefore, some standard unit is needed to provide uniform interpretation of measurements.

EXERCISES

1. Several standard units of length are not commonly known as they either (a) are used only for special types of distances or (b) have been replaced by more recently adopted standard units.

   Use a dictionary or encyclopedia to find a definition of the following standard units.

   a. league
   b. bolt of cloth
   c. light-year
   d. cable
   e. pica
   f. furlong
   g. ell
   h. nautical mile
   i. chain
   j. link
   k. rod

2. Who determines the official standard units of length used in the United States? What is the official U.S. standard of length? (Use a dictionary or encyclopedia.)

HOW LONG IS AN INCH?

In the 14th century, King Edward II of England proclaimed that the English inch should be the length of "three barley-corns, dry, round, and laid end-to-end." He thought that he had finally standardized the inch.

However, there has been disagreement about the inch ever since. Until 1959, the British, United States, and Canadian inches were defined as follows:

British inch - 2.53999560 cm.
Canadian inch - 2.54000000 cm.
U.S.A. inch - 2.54000508 cm.

Although very small, this difference resulted in some World War II supplies requiring precise measurement (guns, planes,...) being custom-made. As a result, some supplies were delayed for months.

In 1959, the United States, England, Australia, Canada, New Zealand, and South Africa agreed to adopt the Canadian inch as their official standard.

ANSWERS

1. a. Usually about 3 miles in length. (varies in some countries and during some periods).
   b. Varies with the material - a bolt of cotton is about [illegible] yards, [illegible] about [illegible].
   c. Distance light travels in one year. (about 5,880,000,000,000 miles).
   d. 720 ft. in U.S. Navy, 608 ft. in British Navy (120 fathoms).
   e. Size of type (12 point — pica type is $\frac{1}{6}$ inch in height) (as on standard pica typewriters).
   f. $\frac{1}{8}$ mile or 660 feet.
   g. 45 inches in England and her colonies.
   h. 6080 ft. in Great Britain.
   6076.1033 ft. in the U.S.A. (essentially equal to one minute of latitude).
   i. 66 ft. - surveyor's chain.
   j. 7.92 in. - one of 100 equal parts of surveyor's chain.
   k. $16\frac{1}{2}$ ft.

2. Congress; meter.

The book by Jeanne Bendick, **HOW MUCH AND HOW MANY?**, is a good resource for exercises 1 and 2.

THINGS TO DISCUSS

1. What would happen if we did not have uniform standard units? If we could not measure precisely?
2. Is there such a thing as an exact measurement? (#2-3, p.25)
3. What disadvantages result from countries differing in their standards of measure? (p.28)
4. Give as many reasons as you can for having a standard length, such as the yard or meter, rather than an arbitary unit of length, such as a piece of rope.

The use of the **EXERCISES** by the entire class is optional. One alternative is to have some selected students report on the results or merely to relate some of the definitions asked for to indicate the variety of units in use. Regardless of the approach selected discuss the ideas indicated under **THINGS TO DISCUSS** with your students.

COMMONLY USED UNITS

SYSTEMS IN USE . . .

Two major systems of measurement are currently in use throughout the world-the English system and the metric system. The metric system is more widely used. With the exception of the United States and Canada, all major countries in the world are presently using the metric system or are in the process of changing to the metric system for use by the general public.

While the centimeter and other units of the metric system are not in common use in this country, they are likely to be in the not-too-distant future.

Some units of length are not used often and their relationships with other units are not worth memorizing (they can be looked up when needed). Following are relationships between some of the basic units of length within the English and metric systems of measurement. Those marked with an asterisk (*) should be memorized, if you have not done so already. These are the ones most commonly used by the general public.

STANDARD UNITS OF LENGTHS

ENGLISH SYSTEM

* 12 inches =1 foot
* 3 feet =1 yard
16.5 feet =1 rod
* 5280 feet =1 mile

METRIC SYSTEM

* 10 millimeters =1 centimeter
10 centimeters =1 decimeter
* 100 centimeters =1 meter
* 1000 meters = 1 kilometer

METRIC OR ENGLISH???

The article on page 30 indicates some of the pros and cons of switching to the metric system in the United States.

COMMONLY USED UNITS

OBJECTIVES

1. Given a list of metric and English units of length the student will be able to rank them from shortest to longest.

2. The student will be able to define the most commonly used metric prefixes.

3. Given a distance to be measured and a list of units, the student shall select the most appropriate unit for measuring that distance (to be developed more fully in Lesson 12).

4. The student shall have immediate recall of the numerical relationship between the basic units of the English and metric system (for those relationships indicated by an asterik (*).

5. To discuss the pros and cons of adopting the metric system for everyday usage.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

1. Rulers
    a. nearest inch
    b. nearest centimeter

B. TEACHER

1. Yardstick
2. Meter stick

CONTENT AND APPROACH

Lesson 5 formally introduces the English and metric system and is designed to develop a feeling for how long the basic units are and how they compare to each other.

Conversion to the Metric System Eyed

By Science Service

WASHINGTON—Conversion to the metric system seems inevitable for the United States, particularly with 90 per cent of the world's people already using it. The question is when and how.

As to when, Sen. Robert P. Griffin, R Mich., who helped draft recently passed federal legislation calling for a three-year study on the ramifications of metric conversion, says that "by 1980 most of the country will be on the metric system."

The question of how to switch is the basic purpose of the three-year study. The secretary of commerce, who will coordinate the study, has been directed to make specific recommendations on overcoming the "practical difficulties" involved.

Many expect the biggest practical difficulty to be the cost of changing over, which most estimates put between $10 billion and $20 billion.

HIGHLY UNLIKELY

General Electric Co. estimates that it would cost them $200 million to convert if they redesigned all their equipment, which is highly unlikely.

However, others say that the relevant question is not "what will it cost to convert" but rather "what will it cost not to."

One economist states that the Stanford Research Institute estimate of $11 billion to change U.S. industry over to the metric system could be paid for in 16 years by the savings made in teaching only the metric system in grade schools.

He says that 25 per cent of the time spent teaching arithmetic could be saved by the elimination of most common fractions and complicated units of measure, and that this would amount to a $705 million a year savings for all pupils across the country.

Heading up the Commerce Department group that will make the three-year study is A.G. McNish, assistant to the director, National Bureau of Standards.

While not wanting to prejudice the results of the study, McNish does not agree that the cost of conversion will run necessarily into the billions.

He says the study will pin down the economic benefits of adopting metric and weigh them against the costs.

McNish says that, from the data obtained, estimates will be made on how fast conversion should go for several industries, so as to keep costs lowest and get the biggest benefits, primarily increased foreign trade.

Dr. John F. Kincaid, assistant secretary of commerce for science and technology, points out that the metric conversion issue has a critical bearing on the amount of the country's foreign trade and its balance of payments.

He believes, along with a number of economists, that until the U.S. switches, it will be denied much of the world market that is developing as the emerging nations industrialize, a market that Dr. Kincaid says will reach "almost astronomical proportions."

In addition, the study will explore the costs and problems of teaching the metric system in the country's schools, and of changing consumer transactions to metric, such as buying milk by the liter, meat by the kilogram or carpeting by the meter.

When the study is completed, the fireworks will begin. It will be up to Congress to decide specifically how to convert. This will involve the potentially explosive question of the extent to which metric adoption will be mandatory, rather than voluntary.

STAGED BASIS

For example, the British have begun mandatory conversion to metric in a staged basis so that by 1975 every segment of the country will be on metric.

McNish says that he doubts very much that this is the way it will go for the U.S.

He thinks industry would not tolerate a mandatory approach, although he believes federal tax incentives to ease the cost burden where it exists may be provided.

He says it is probable that only consumer transactions would be covered by mandatory federal legislation. And as for teaching metric in the schools, McNish sees this as a state and local matter.

This article appeared in the Pontiac Press, Pontiac, Michigan, November 6, 1968.

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Additional reference sources for the discussion of the article on page 30 are listed under RESOURCE MATERIALS on pages xvi-xvii at the beginning of this booklet. This article is included to provide a rationale for the study of the metric system.

You may prefer to postpone the class discussion of this article and/or some of the articles listed under RESOURCE MATERIALS until the students have studied the metric system more extensively.

The question of whether or not to switch to the metric system for everyday usage is not a hypothetical question--it is one that is currently being debated and will affect all of us.

Following are some sample questions which could be used to motivate a class discussion of this article.

1. What advantages are stated or implied in the article (p. 30) for switching from our present English system of measurement? What disadvantages?

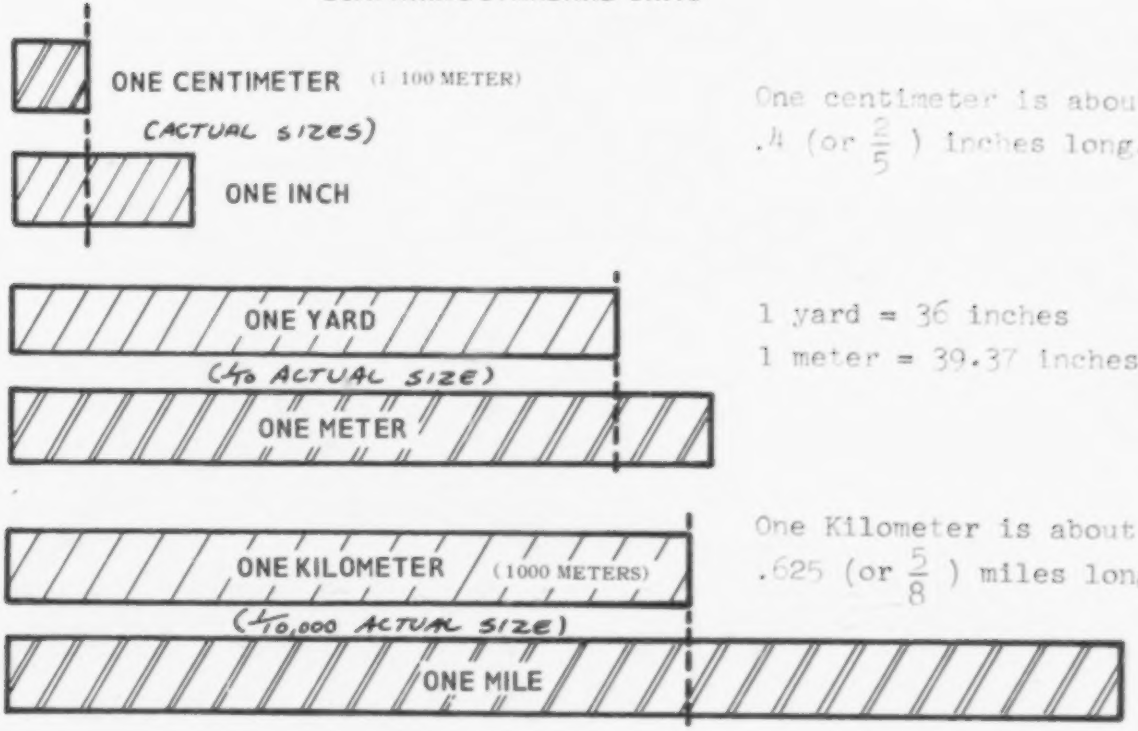
2. Consider the following statement.

   > "The metric system is harder to use than the English system. I studied the metric system last year in school and never did understand it."

   Is the above statement a valid argument for not switching to the metric system? Why or why not?

3. Suppose the English system of measurement was completely abolished and you were allowed to use only the metric system. What changes would affect you?

COMPARING STANDARD UNITS


ONE CENTIMETER (1/100 METER)
(ACTUAL SIZES)
ONE INCH
ONE YARD
(1/10 ACTUAL SIZE)
ONE METER
ONE KILOMETER (1000 METERS)
(1/10,000 ACTUAL SIZE)
ONE MILE

One centimeter is about .4 (or $\frac{2}{5}$) inches long.

1 yard = 36 inches
1 meter = 39.37 inches.

One Kilometer is about .625 (or $\frac{5}{8}$) miles long.

The above scale drawings indicate the relative lengths of some comparable English and metric units of length. Notice that the drawings are drawn to different scales.

✓ POINT

1. Circle the longer unit in each pair.
    a. Mile Kilometer
    b. Yard Meter
    c. Inch Centimeter
    d. Mile Meter

Use the drawings on page 91, a meter stick, a yardstick, and provide each student with rulers graduated in whole inches and centimeters to aid the student in visualizing the comparison between metric and English units.

ANSWERS

1. a. Mile
2. b. Meter
3. c. Inch
4. d. Mile

These responses should have been circled.

PREFIXES AND THE DECIMAL SYSTEM . . .

The metric system is convenient to use because all units of length are related to the basic unit, the meter, in the same way that the place values (thousandths, hundredths, tenths, tens, hundreds,...) of the decimal system of numeration are related to the units ... (or ones) place.

For example, a centimeter is $\frac{1}{100}$ of a meter and a kilometer is 1000 meters. The relationship between the metric units is determined by using the metric prefixes.

METRIC PREFIXES

milli- $\frac{1}{1000}$	(.001)	deca- 10
centi- $\frac{1}{100}$	(.01)	hecto- 100
deci - $\frac{1}{10}$	(.1)	kilo- 1000

The prefixes underlined are those most commonly used.

These same prefixes are used with the basic metric units of mass (gram) and capacity (liter).

In contrast, the relationships between English units of length are not as consistent.

✓POINT

1. Give the meaning of the following metric prefixes.

   a. milli ______
   b. kilo ______
   c. centi ______

The text on page [illegible] indicates the similarity between the relationship of the metric prefixes to the meter and the decimal place values to the units (or ones) place.

thousands	hundreds	tens	units	tenths	hundredths	thousandths
1000	100	10	1	.1 (1/10)	.01 (1/100)	.001 (1/1000)
kilo	hecto	deca	basic unit	deci	centi	milli

Suppose all metric lengths were written in terms of the meter. Consider the length 47.68 meters.

$$47.68 = (4 \times 10\text{ m}) + (7 \times 1\text{ m}) + (6 \times .1\text{ m}) + (8 \times .01\text{ m})$$
$$= 4\ \underline{\text{deca}}\text{meters} + 7\text{ meters} + 6\ \underline{\text{deci}}\text{meters} + 8\ \underline{\text{centi}}\text{meters.}$$

Notice the similarity between the expanded form above and the expanded form if only the decimal places were considered. Another type of similarity which can be noted is:

$$47.68 = \frac{4768}{100} = 4768 \times \frac{1}{100}$$

$$\text{Therefore, } 47.68\text{ m} = 4768 \times \frac{1}{100}\text{ m} = 4768\text{ cm.}$$

This exposure and use of metric prefixes is too limited in scope for the student to fully appreciate the simiplicity of these decimal relationships.

EXERCISES

1. Circle the longer unit in each pair.

   a. meter yard
   b. centimeter inch
   c. centimeter meter
   d. kilometer mile
   e. centimeter millimeter
   f. foot yard
   g. meter foot
   h. inch millimeter

2. Complete:

   a. 1 mile = _____ feet
   b. 1 meter = _____ millimeters
   c. 1 meter = _____ centimeters
   d. 1 centimeter = _____ millimeters

3. Suppose that the metric system, with the prefixes, milli, deci, centi, and kilo, was used to measure time.

   a. If your class period was a deciday long, would you like it better? Why or why not? ________________

      ____

      ____

   b. About how many centidays would it take to make an hour? __________
   c. A kiloday would be about how many years? _________

4. What is a more common name for a centidollar? a decidollar?

   __________ __________

5. Arrange the following units in order of length from smallest to largest. (Indicate the smallest by 1, next smaller by 2,...)

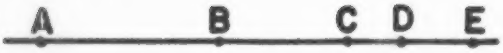
   _____ inch
   _____ kilometer
   _____ yard
   _____ millimeter
   _____ foot
   _____ meter
   _____ mile
   _____ centimeter

Exercises 1, 5 , 6, 7 and 8 give the student practice in visualizing relative sizes. Exercises 2, 3 and 4 drill on metric prefixes and numerical relationships between the basic units.

ANSWERS

EXERCISES

1. a. meter
   b. inch
   c. meter
   d. mile
   e. centimeter
   f. yard
   g. meter
   h. inch
2. a. 5,280
   b. 1,000
   c. 100
   d. 10
3. a. 1 deciday = 2.4 hours = 2 hours 24 min.
   The answers to this question should be interesting.
   b. A little more than 4. $(4\frac{1}{6})$
   c. About 3 years.
4. penny; dime
5. 3 inch
   7 kilometer
   5 yard
   1 millimeter
   4 foot
   6 meter
   8 mile
   2 centimeter

6. 

a. Name the segment which you believe is closest to an inch in length. ________

b. Name the segment which you believe is closest to a centimeter in length. __________

7. Circle the unit which best completes the statement.

a. The longest unit is...

a centimeter a meter an inch

b. The length of a room would be measured in...

centimeters kilometers meters

c. The distance from Detroit to Chicago would be measured in...

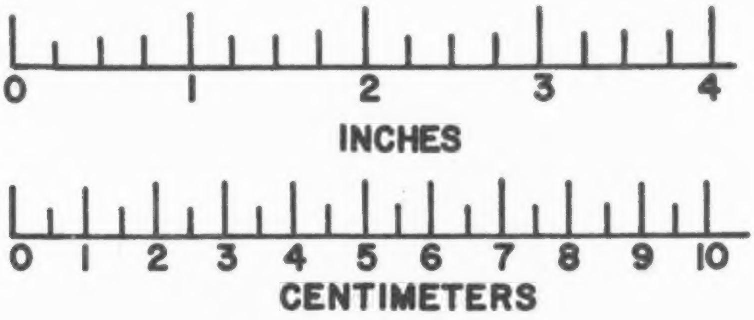
kilometers millimeters meters

d. Your height would be measured in...

millimeters meters centimeters

e. 100 centimeters equals...

1 kilometer 1 meter 1 yard



8. Comparing the inch and centimeter scales above, complete the following.

a. 5 cm. ≈ _____ in. c. 20 cm. ≈ _____ in.

b. 3 in. ≈ _____ cm.

(~ is read "approximately the same as".)

ANSWERS

6. a. $\overline{AB}$
   b. $\overline{DF}$

7. a. meter
   b. meters
   c. kilometers

   d. centimeters (possibly meters)
   e. 1 meter

8. a. 5 cm. $\approx$ 2 in.
   b. 3 in. $\approx$ 7.5 cm.
   c. 20 cm. $\approx$ 8 in.

MORE OF THE SAME

The previous lesson (pp.29-34) introduced the more commonly used English and metric units of length and the metric prefixes. The exercises in this lesson continue the development of these topics. Work the exercises carefully, referring to the information given. Ask for assistance if there are problems which give you difficulty.

EXERCISES

Refer to the information below for questions 1 and 2.

UNITS OF LENGTH

English System	Metric System
12 inches = 1 foot	10 millimeters = 1 centimeter
3 feet = 1 yard	100 centimeters = 1 meter
5280 feet = 1 mile	1000 meters = 1 kilometer

1. Write the ratios of the following English units.

____ a. 1 foot to 1 inch ($\frac{\text{1 foot}}{\text{1 inch}} = \frac{\text{12 inches}}{\text{1 inch}} = \frac{?}{?}$)

____ b. 1 yard to 1 foot

____ c. 1 inch to 1 yard

____ d. 1 mile to 1 foot

2. Write the ratios of the following metric units.

____ a. 1 centimeter to 1 millimeter ($\frac{\text{1 cm.}}{\text{1 mm.}} = \frac{\text{10 mm.}}{\text{1 mm.}} = \frac{?}{?}$)

____ b. 1 centimeter to 1 meter

____ c. 1 meter to 1 millimeter

____ d. 1 kilometer to 1 meter.

MORE OF THE SAME

OBJECTIVES

This lesson provides additional practice on the topics introduced in Lesson 5. Use the same list of objectives as those given for Lesson 5 (See page T 29).

CONTENT AND APPROACH

One indication of the simplicity of the metric system as opposed to the English system is the constant ratio of 1 to 10 between each unit and the next larger unit; thus the ratio of any two metric units is a power of 10.

Exercises 1 and 2 provide practice in writing ratios and are a first attempt to indicate this relationship between units. The value of such a constant ratio, however, can not be shown with two exercises and can only be developed through practice and use of the metric system.

ANSWERS

1. a. $\frac{12}{1}$

   b. $\frac{3}{1}$

   c. $\frac{1}{36}$

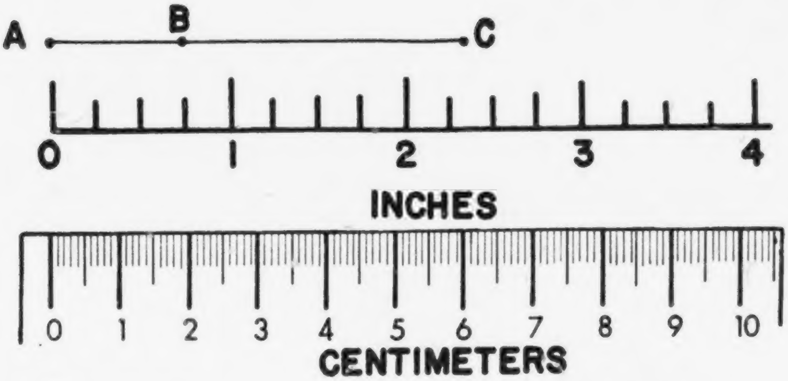
   d. $\frac{5280}{1}$

2. a. $\frac{10}{1}$

   b. $\frac{1}{100}$

   c. $\frac{1000}{1}$

   d. $\frac{1000}{1}$



Refer to the above illustration for exercises 3-7.

3. Each inch on the top ruler is divided into four equal parts. Each part is what fractional part of an inch? ____

4. Each centimeter on the bottom ruler is divided into ten equal parts. What is the name of each of these parts? _____

5. Give the measures of the following segments to the nearest centimeter (cm.).

   a. $m(\overline{AB})$ = ____ cm.   c. $m(\overline{BC}) = m(\overline{AC}) - m(\overline{AB})$ = ____ cm.

   b. $m(\overline{AC})$ = ____ cm.

6. A 35 mm. camera uses film that is 35 millimeters (3.5 cm.) wide. What is the width of this film to the nearest $\frac{1}{4}$ inch? ___________

7. To obtain a wide-screen effect, commercial theaters use motion-picture film that has a width of 70 mm.

   a. How many centimeters wide is this film? _____

   b. What is the width of this film to the nearest $\frac{1}{4}$ inch?

   ____

8. Electricity for lights and other electrical applicances is measured by an electric meter. The unit of measure is the watt-hour or kilo-watt hour (kwh), which is 1000 watt-hours. (kilo = 1000)

An appliance rated 40 watts will use 40 watts of electricity per hour.

Exercise 5 is not directly related to the topics under development. However, the questions review the skill of measuring to the nearest unit.

The use of an arbitrary "zero point" for measuring (5c) will be developed in Lesson 9, (pp. 58 - 66).

AMSWERS

3. $\frac{1}{4}$

4. millimeter

5. a. $m(\overline{AB}) = \underline{2}$ cm. c. $m(\overline{BC}) = \underline{4}$ cm.
   b. $m(\overline{AC}) = \underline{6}$ cm.

6. $1\frac{1}{4}$ in.

7. a. 7 cm.
   b. $2\frac{3}{4}$ in.

For example, a 1000-watt heater will use 1 kilowatt hour of electricity in 1 hour (1000 watts x 1 hr. = 1000 watt-hours = 1 kwh). In 10 hours, a 100-watt bulb will also use 1 kwh of electricity (100 watts x 10 hrs. = 1000 watt - hours = 1 kwh).

a. How many hours will it take for two 100-watt bulbs to use 1 kwh of electricity? ________

b. An electric fan is marked "40 watts." How many hours will it take to use 2 kwh of electricity? ________

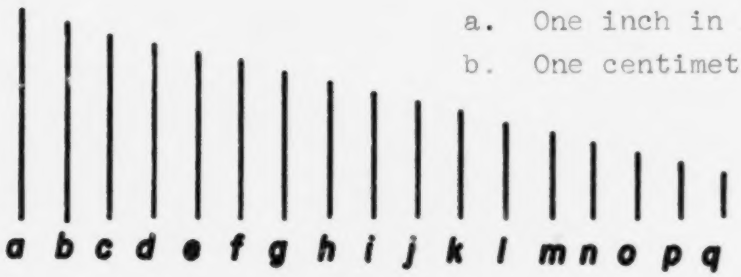
9. Circle the best estimate in each exercise.

a. Height of a man	1 meter	2 meters	100 centimeters
b. Length of a School Bus	25 yards	18 feet	18 yards
c. Length of a dollar bill	6 inches	10 inches	12 inches

10. Indicate the letter of the segment which you believe is nearest to...

a. One inch in length. __________

b. One centimeter in length. ________



*11. Metric prefixes are used in conjunction with several units of measures and objects. For example the prefix mega means 1,000,000 and a megaton is 1,000,000 tons. What is the meaning of the following words?

a. decade
b. decathlon
c. micrometer
d. centigrade
e. mill

ANSWERS

8. a. 5 hours
   b. 50 hours

9. The following responses should be circled.

   a. 2 meters
   b. 18 yards
   c. .6 inches

10. a. d
    b. c

11. This exercise is optional and illustrates the wide use of metric prefixes. A dictionary will be needed for this exercise.

    a. A period of 10 years.
    b. An Olympic track event consisting of ten separate events.
    c. A precision measuring instrument for measuring very small distances. (The more commonly used ones measure to the nearest .001 inch.)
    d. Divided into 100 parts, as on a centigrade thermometer scale.
    e. A unit of money, usually used in relation to tax rates, equal to $\frac{1}{1000}$ of a dollar or $\frac{1}{10}$ of a cent.

✓POINT

Use the following questions to check your understanding of Lessons 4-6. If you have difficulty with any questions, it may indicate a topic you should review.

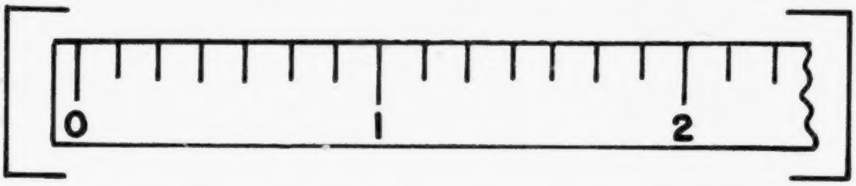
1. For each pair circle the phrase that indicates the more precise measurement.

To the nearest... a. .1 in. .1 cm.

b. yard meter

c. $\frac{1}{8}$ in. $\frac{1}{16}$ in.

2. The ruler below is graduated in ____________ of a unit. (what fractional part?)



3. Define the following metric prefixes.

a. centi c. kilo

b. milli

4. Complete:

a. _____ centimeters = 1 meter

b. 1 centimeter = ____ millimeters

c. 1 yard = ____ feet

d. 1 in. ≈ ____ cm.

5. Arrange each set of units in order from largest to smallest.

a. centimeter inch millimeter

b. yard foot meter

c. inch meter mile kilometer

ANSWERS

1. To the nearest...
   a. .1 cm.
   b. yard
   c. $\frac{1}{16}$ inch

2. sevenths

3. a. centi = $\frac{1}{100}$
   b. milli= $\frac{1}{1000}$
   c. kilo = 1000

4. a. 100 centimeters = 1 meter
   b. 1 centimeter = 10 millimeters
   c. 1 yard = 3 feet
   d. 1 inch ≈ $2\frac{1}{2}$ centimeters

5. a. inch, centimeter, millimeter
   b. meter, yard, foot
   c. mile, kilometer, meter, inch

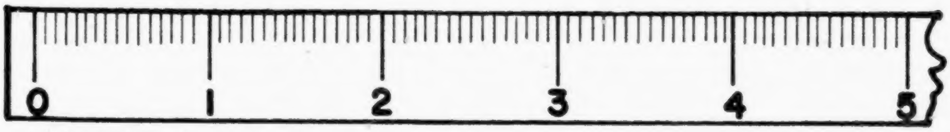
ORIGIN OF THE MILE. . .

The mille or mile, as used by the Romans, was defined as 1,000 paces (double steps). The Latin words for 1,000 paces were millia passuum, later translated into English as mile.

About the year 1500, the mile was changed to 5,280 feet. This was done so that a mile could be divided into exactly 8 furlongs, the most common units of length for land measure at that time.
(1 furlong = 40 rods = 660 feet)

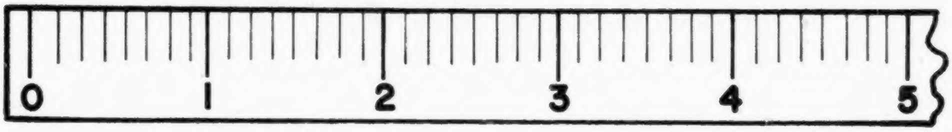
$$\begin{array}{r} 8 \\ 660\overline{)\,5280} \\ -5280 \\ \hline 0 \end{array}$$

A.



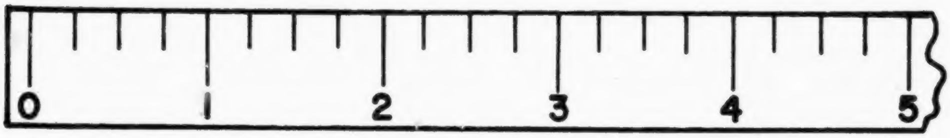
SIXTEENTHS OF AN INCH

B.



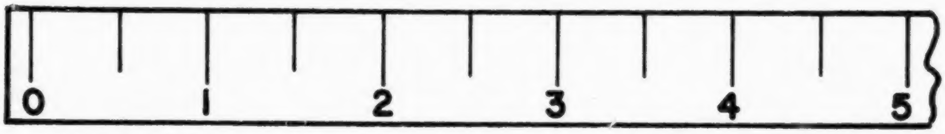
EIGHTHS OF AN INCH

C.



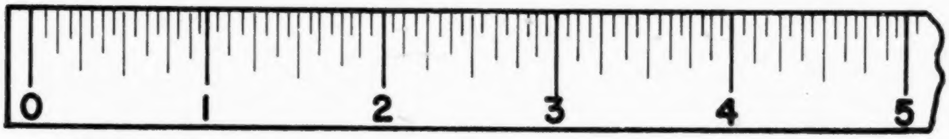
FOURTHS OF AN INCH

D.



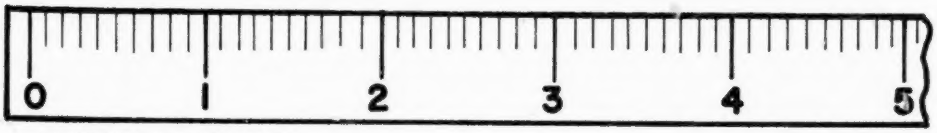
HALF-INCHES

E.



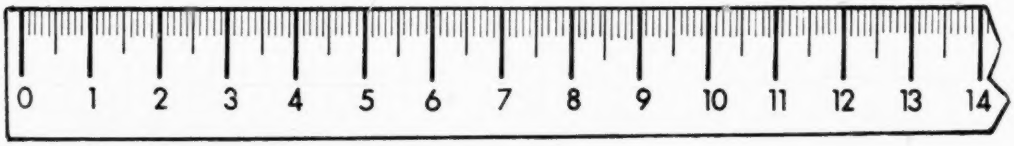
SIXTEENTHS OF AN INCH

F.



TENTHS OF AN INCH

G.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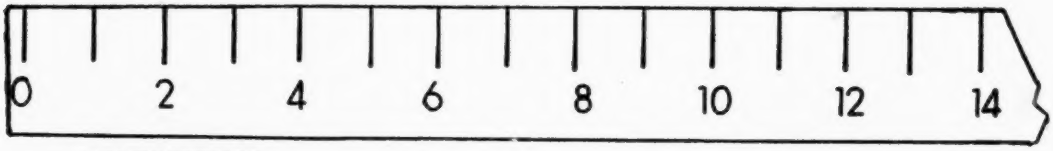
TENTHS OF A CENTIMETER

H.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CENTIMETERS

WHAT IS THE SCALE?

OBJECTIVES

1. Using rulers graduated in subdivisions of an inch ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{10}, \frac{1}{16}$) as a guide, the student shall be answer questions such as...

   a. If $\frac{3}{4} = \frac{x}{8}$, then $x =$ ___.

   b. If $\frac{1}{2} + \frac{1}{8} = \frac{a}{8} + \frac{1}{8} = \frac{c}{8}$, then $a =$ ___ and $c =$ ___.

2. Given rulers graduated like those pictured on page 40 and an indicated length such as 3.7 in. or $2\frac{5}{8}$ in., the student shall indicate the corresponding location on the ruler scale.

3. The student shall be able to answer questions such as...

   For each ruler location circle the whole unit measure to which it is closest. If it is equally close to both, circle both.

Ruler Location	Whole Units
5.7 cm.	5 cm. 6 cm.
$2\frac{9}{16}$ in.	2 in. 3 in.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

*1. Rulers graduated in the following subdivisions of an inch and centimeter:

a. cm.

b. .1 cm.

c. $\frac{1}{2}$ in.

d. $\frac{1}{4}$ in.

e. $\frac{1}{8}$

f. $\frac{1}{10}$ in.

g. $\frac{1}{16}$ in. (markings of equal length)

h. $\frac{1}{16}$ in. (markings of unequal length)

WHAT IS THE SCALE?

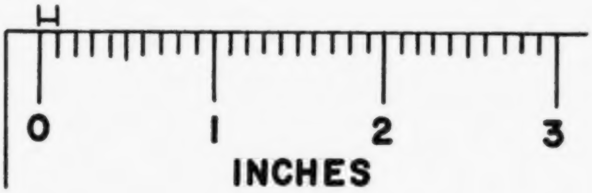
You will be given a packet of rulers like those pictured on page 40. These rulers will be used in the remainder of this booklet.

The markings of ruler E are the most commonly used for every-day purposes. You must understand what the marks of different length mean before you can use a ruler accurately.

THE SCALE IS . . .

Before a ruler can be used to measure distances, its scale must be determined. The scale depends on the length of the ruler's smallest subdivision.

For example, each inch of the ruler pictured below is divided into ten equal parts.



Therefore, each of the small subdivisions on the above ruler (one subdivision is indicated above the ruler) represents a length of $\frac{1}{10}$ inch. We say that the ruler has a scale graduated in tenths of an inch.

EQUIPMENT AND TEACHING AIDS (Cont'd)

B. TEACHER

1. Overhead projector.
2. Projection screen.
3. Acetate rulers (graduated like those provided for the student).

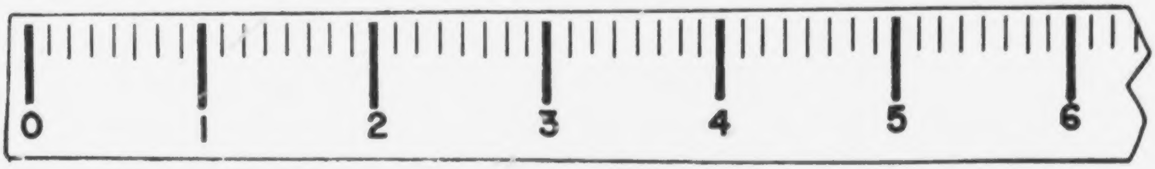
CONTENT AND APPROACH

Before any ruler can be used, its scale and the meaning of its markings must be determined. The scale of a ruler depends on the length of its smallest subdivision. Earlier exercises related to this lesson include exercise # 1, p.24. The student is provided with a packet of cardboard rulers graduated like those pictured on page 40. These rulers will be used in Lesson 7 and throughout the remainder of the booklet. The acetate rulers can be used with an overhead projector to provide further illustrations and examples for your students if needed.

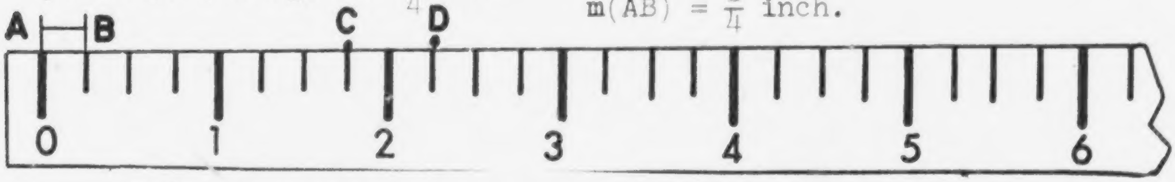
The cardboard rulers provided have been purposely designed with the zero not at the end. This gives the student the idea that markings are more accurate to use than the end of the ruler because corners soon become worn, thus introducing error (See Guideline # 1, p.47).

Rulers with several different scales have been provided at this stage because too many subdivisions and/or markings of different length tend to be confusing to the student who is not yet proficient in the use of a ruler. Eventually (do not rush them), the students should be able to use a ruler such as ruler E (p.40) to measure to the nearest in., $\frac{1}{2}$ in., $\frac{1}{4}$ in., $\frac{1}{8}$ in., or $\frac{1}{16}$ in.

The ruler below is graduated in eighths of an inch. (How can you tell?) It could be used to measure lengths to the nearest eighth of an inch.



Each of the smaller subdivisions on the ruler below represents a length of $\frac{1}{4}$ inch. $m(\overline{AB}) = \frac{1}{4}$ inch.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Point C on the scale represents 7 one-fourth inches.

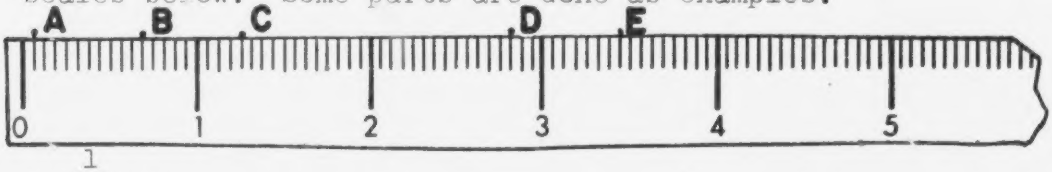
7 one-fourth inches = $7 \times \frac{1}{4}$ inches = $\frac{7}{4}$ in. = $1\frac{3}{4}$ in.

Point D represents how many inches?

✓ POINT

1. Give the meaning for each labeled point on the ruler scales below. Some parts are done as examples.

a.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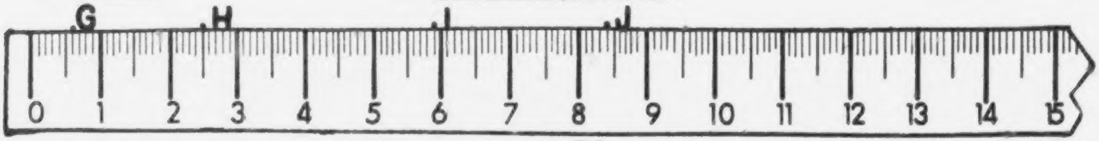
A: $\frac{1}{16}$ in.

B: _____

C: _____

D: $\frac{45}{16}$ or $2\frac{13}{16}$ in.

E: _____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15

G: _____

H: _____

I: 5.9 cm. ($\frac{59}{10}$ cm.)

J: _____

CONTENT AND APPROACH

After the scale of a ruler (length of the smallest subdivision) has been determined, the student should be able to give the meaning of any indicated point of the ruler scale. Notice that each marking is interpreted as a multiple of the scale. For example, the scale on the second ruler pictured on page 42 is graduated in fourths of an inch. Point D represents 9 one-fourth inches = $9 \times \frac{1}{4}$ in. = $\frac{9}{4}$ in. = $2\frac{1}{4}$ in.

If your students have difficulty converting improper fractions to mixed numbers (and vice-versa), provide additional practice prior to their using the ✓POINT to test their understanding of pages 41 and 42. Some strategies for explaining this process were outlined in comments on LESSON 1.(See **CONTENT AND APPROACH** , page T 7).

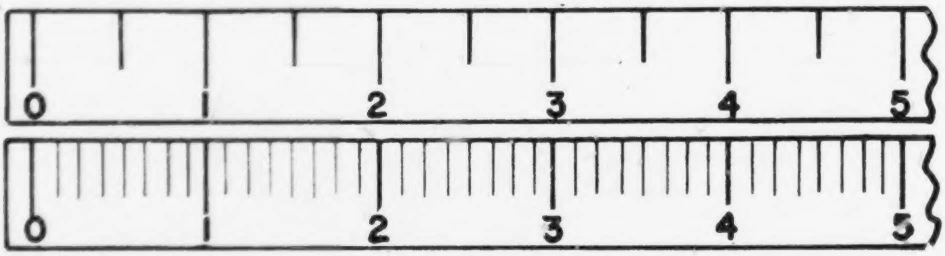
ANSWERS (✓POINT)

1. A: $\frac{1}{16}$ in. C: $\frac{20}{16}$ or $1\frac{4}{16}$ in. E: $\frac{55}{16}$ or $3\frac{7}{16}$ in.
   B: $\frac{11}{16}$ in. D: $\frac{45}{16}$ or $2\frac{13}{16}$ in.

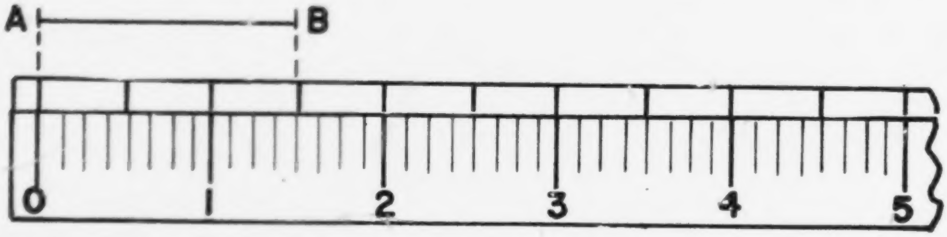
2. G: .6 cm. I: 5.9 cm. ($\frac{59}{10}$ cm.)
   H: 2.5 cm. J: 8.4 cm. ($\frac{84}{10}$ cm.)

EQUIVALENT RULER LENGTHS

Ruler scales can also be used to indicate fractions that are equivalent (have the same value). For example, suppose two rulers graduated in half inches and eighths of an inch are used.



Place the two ruler scales next to each other.



Depending on the ruler used.

$m(\overline{AB}) = 3$ one-half inches

$= 3 \times \frac{1}{2}$ in.

$m(\overline{AB}) = \frac{3}{2}$ in. $= 1\frac{1}{2}$ in.

or

$m(\overline{AB}) = 12$ one-eighth inches

$= 12 \times \frac{1}{8}$ in.

$m(\overline{AB}) = \frac{12}{8}$ in. $= 1\frac{4}{8}$ in.

These results indicate that $\frac{3}{2} = \frac{12}{8}$ (How?)

The truth of this result could be checked by the cross-product method which was developed in ACTIVITIES WITH RATIO AND PROPORTION.

$\frac{3}{2} = \frac{12}{8}$

because

$3 \times 8 = 2 \times 12$

✓ POINT

Give the missing numbers so that the fractions for each part are equivalent. (Your rulers may be used.)

1. $\frac{\square}{16} = \frac{7}{8}$
2. $\frac{\square}{4} = \frac{4}{16}$
3. $\frac{3}{\square} = \frac{12}{8}$
4. $\frac{10}{8} = \frac{\square}{4}$

CONTENT AND APPROACH

Equivalent fractions and the cross product method were developed in a previous booklet (**ACTIVITIES WITH RATIO AND PROPORTION** , Lessons 3 and 8). This prior exposure and development is assumed and this section is intended as a review of equivalent fractions and indicates the use of rulers as guides in determining some of the more commonly used equivalents.

If your students need additional review or development of these two topics, refer to the strategies outlined in the previous booklet and use the acetate rulers to provide additional examples.

ANSWERS (✓ POINT)

1. $\frac{14}{16} = \frac{7}{8}$

2. $\frac{1}{4} = \frac{4}{16}$

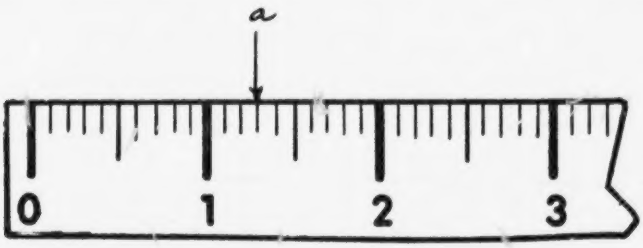
3. $\frac{3}{2} = \frac{12}{8}$

4. $\frac{10}{8} = \frac{5}{4}$

NOTE: Allowance for small variations in readings may need to be made due to scale distortions in the cardboard and acetate rulers.

EXERCISES

1. The ruler below is graduated in tenths of an inch.



Indicate by a letter and arrow each of the following locations on the above ruler scale. Part a is done as an example.

a. $1\frac{3}{10}$ in.

b. 2.5 in. ($2\frac{5}{10}$ in.)

c. 3.2 in.

d. $\frac{7}{10}$ in.

e. 1.9 in.

f. .2 in.

You may refer to the ruler in #1 for questions 2 and 3.

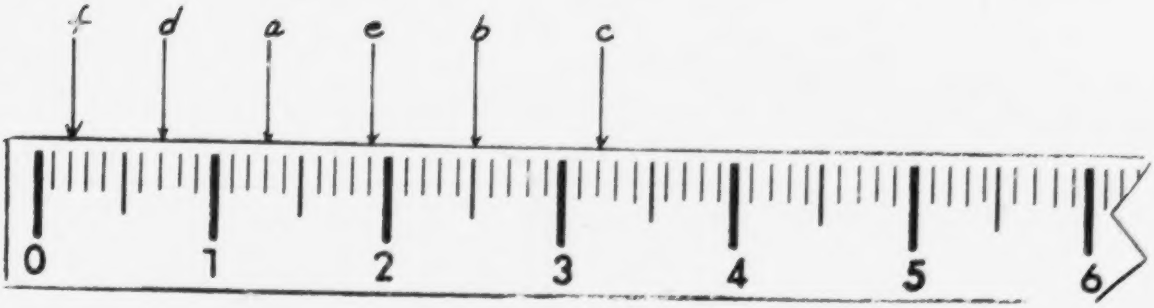
2. a. How many tenths in one inch? _____
   b. How many tenths in 2 inches? _____
   c. How many tenths in a half-inch? _____
   d. Is .4 inch more or less than a half-inch? _____
   e. Is 2.8 inches more or less than $2\frac{1}{2}$ inches? _____

3. For each ruler location, circle the whole inch measure to which it is closest. If it is equally close to both, circle both.

	RULER LOCATION	WHOLE INCHES	
a.	1.4 in.	1 in.	2 in.
b.	.7 in.	0 in.	1 in.
c.	.3 in.	0 in.	1 in.
d.	1.5 in.	1 in.	2 in.
e.	2.3 in.	1 in.	3 in.

ANSWERS

1.



2. a. 10 tenths in one inch.

b. 20 tenths in two inches.

c. 5 tenths in a half-inch.

d. .4 in. is less than a half-inch.

e. 2.8 in. is more than $2\frac{1}{2}$ inches.

3. a. 1 inch

b. 1 inch

c. 0 inches

d. 1 inch; 2 inches

e. 3 inches

4. Give the numbers which when substituted for the letters will make the fractions for each part equivalent. Write your answers in the blanks provided.

a $\frac{5}{8} = \frac{x}{16}$, x = ________ b. $\frac{c}{4} = \frac{6}{8}$, c = ________

If $\frac{5}{8} = \frac{x}{16}$, then x = ________ If $\frac{c}{4} = \frac{6}{8}$, then c = ____

c. If $\frac{a}{16} = \frac{3}{4}$, then a = ________ d. If $\frac{b}{16} = \frac{3}{8}$. then b = ____

e. If $2\frac{x}{8} = 2\frac{1}{2} = 2\frac{y}{10}$, then x = ________and y = ________

f. If $\frac{28}{16} = \frac{r}{4} = 1\frac{s}{4}$, then r = ________and s = ________

5. Give the numbers which when substituted for the letters will make each of the following statements true. Write your answers in the blanks provided.

a. If $\frac{3}{4} + \frac{1}{16} = \frac{x}{16} + \frac{1}{16} = \frac{y}{16}$, then x = ________and y = ________.

b. If $\frac{1}{2} + \frac{3}{8} = \frac{c}{8} + \frac{3}{8} = \frac{d}{8}$, then c = ________and d = ________.

c. If $\frac{3}{16} + \frac{5}{8} = \frac{3}{16} + \frac{r}{16} = \frac{s}{16}$, then r = ________and s = ________.

d. If $\frac{3}{4} + \frac{5}{8} = \frac{a}{8} + \frac{5}{3} = \frac{b}{8} = 1\frac{c}{8}$, then a = ____, b = ____, and c =____

e. If $\frac{7}{8} - \frac{1}{2} = \frac{7}{8} - \frac{m}{8} = \frac{n}{8}$, then m = ______ and n = ______.

f. If $\frac{1}{4} + \frac{3}{8} = \frac{t}{8}$, then t = ______.

g. If $\frac{5}{16} + \frac{3}{8} = \frac{a}{b}$, then a = _____and b = ______.

h. If $\frac{9}{10} - \frac{1}{2} = \frac{j}{k}$, then j = _____ and k = _____.

ANSWERS

4. a. $x = \underline{10}$ b. $c = \underline{3}$

c. $a = \underline{12}$ d. $b = \underline{6}$

e. $x = \underline{4}$ and $y = \underline{5}$.

f. $r = \underline{7}$ and $s = \underline{3}$.

5. a. $x = \underline{12}$ and $y = \underline{13}$.

b. $c = \underline{4}$ and $d = \underline{7}$.

c. $r = \underline{10}$ and $s = \underline{13}$.

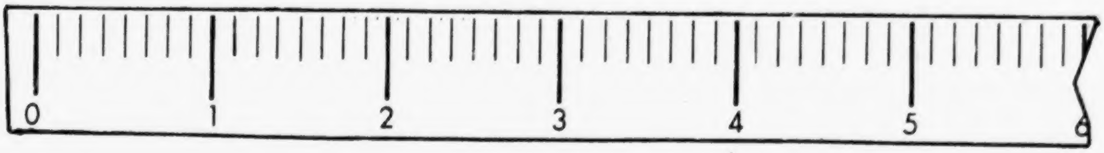
d. $a = \underline{6}$, $b = \underline{11}$, and $c = \underline{3}$

e. $m = \underline{4}$ and $n = \underline{3}$

f. $t = \underline{5}$.

g. $a = \underline{11}$ and $b = \underline{16}$.

h. $j = \underline{4}$ and $k = \underline{10}$. (If student writes answers in simplest form, $j = \underline{2}$ and $k = \underline{5}$.)



6. Indicate by a letter and arrow each of the following locations on the above ruler scale.

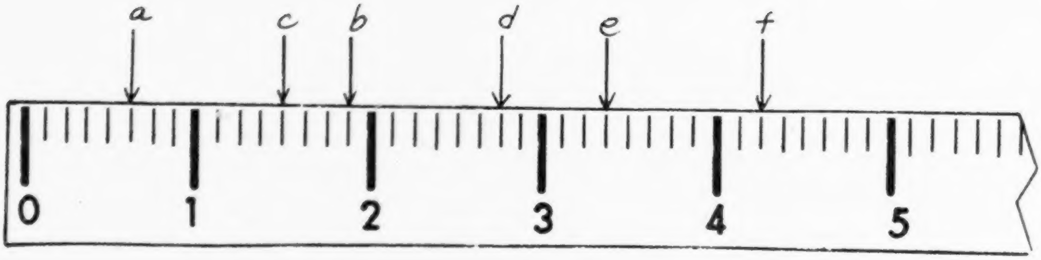
a. $\frac{5}{8}$ in.	d. $2\frac{3}{4}$ in.
b. $1\frac{7}{8}$ in.	e. $3\frac{3}{8}$ in.
c. $1\frac{1}{2}$ in. ($1\frac{4}{8}$ in.)	f. $4\frac{1}{4}$ in.

7. For each ruler location, circle the whole inch measure to which it is closest. If it is equally close to both, circle both.

RULER LOCATION	**WHOLE INCHES**	
a. $1\frac{5}{8}$ in.	1 in.	2 in.
b. $\frac{3}{8}$ in.	0 in.	1 in.
c. $4\frac{3}{4}$ in.	4 in.	5 in.
d. $2\frac{1}{2}$ in.	2 in.	3 in.
e. $3\frac{7}{8}$ in.	4 in.	3 in.

ANSWERS

6.


a
c
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d
e
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7. a. 2 in.

   b. 0 in.

   c. 5 in.

   d. 2 in., 3 in.

   e. 4 in.

RULES TO LIVE BY

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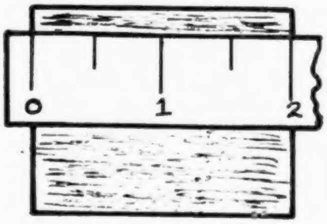
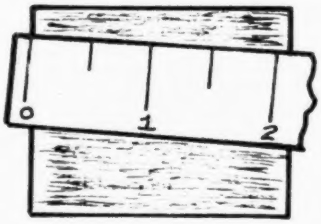
GUIDE LINES FOR RULER USE

1. Use rulers that have square, clean-cut edges. A ruler that is warped, rounded, or worn will give inaccurate results despite careful use.

2. When measuring the length or width of a rectangular object, measure parallel and close to the edge being measured.



CORRECT

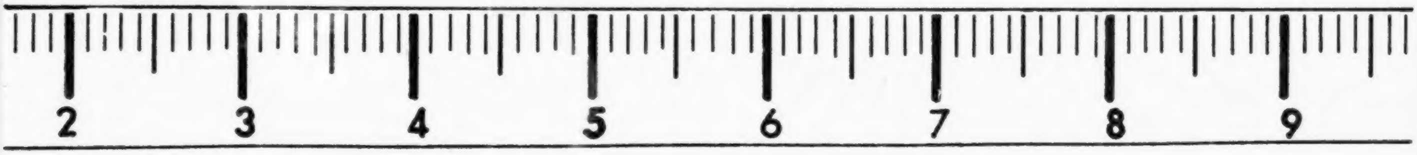


INCORRECT

3. Use a unit of length suitable for the distance being measured and the purpose for which it is being used. (This idea will be further developed in Lesson 12.)

4. Know the scale used on the ruler before reading the measurement.

5. For best results, place the ruler in such a position so that the scale is as close as possible to the object being measured.



RULES TO LIVE BY

OBJECTIVES

1. The student shall be able to use a ruler to measure to the nearest...

   (a) $\frac{1}{8}$ in. (b) $\frac{1}{16}$ in. (c) .1 in.

   (d) .1 cm.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

*1. Packet of rulers - use will be made of most of the rulers provided for Lesson 7.

B. TEACHER

1. Overhead projector.
2. Projection screen.
*3. Prepared transparencies.
*4. Acetate rulers (graduated like those used by students).
*5. Acetate T-square overlay.
*6. Acetate overlay (to check exercise 2, p.52).

ADDITIONAL COMMENT ON GUIDELINE NUMBER 5

It is good practice to place the ruler in such a position that the scale is as close as possible to the object being measured; otherwise the thickness of the ruler causes parallax errors (variations in reading due to the angle from which the reading is taken). Use the pictures reproduced on TR 8-1 if you wish to discuss this point with your class.

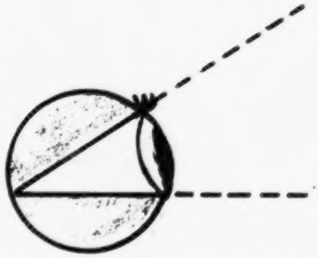
COMPETENCE ASSUMED.....

When lengths are measured, the ability to use the measuring instrument properly is usually assumed. However, carelessness and lack of knowledge may cause any measuring instrument to be used improperly. Many mistakes can be easily corrected by following some basic guidelines in the proper use of a ruler. (See page 47.)

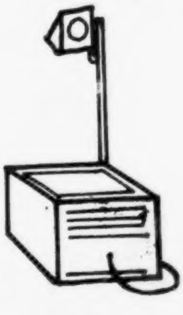
REDUCING ERRORS. . ..

Even with careful use, small "errors" occur. Some things which cause errors are: The user is tired, the ruler slipped, the ends of the distance being measured are not lined up carefully with the ruler scale, or the ruler is warped or rounded. Reading a ruler involves sighting, estimating, and judgment. All three tend to give an approximate value to the measurement, resulting in small "errors". We cannot hope to eliminate all errors but with practice, the size of these errors can be reduced.

WATCH CAREFULLY. . .



The remainder of this lesson will be developed with the aid of the overhead projector. Listen carefully and participate in the discussion. If you have questions during the discussion, ask them.



CONTENT AND APPROACH

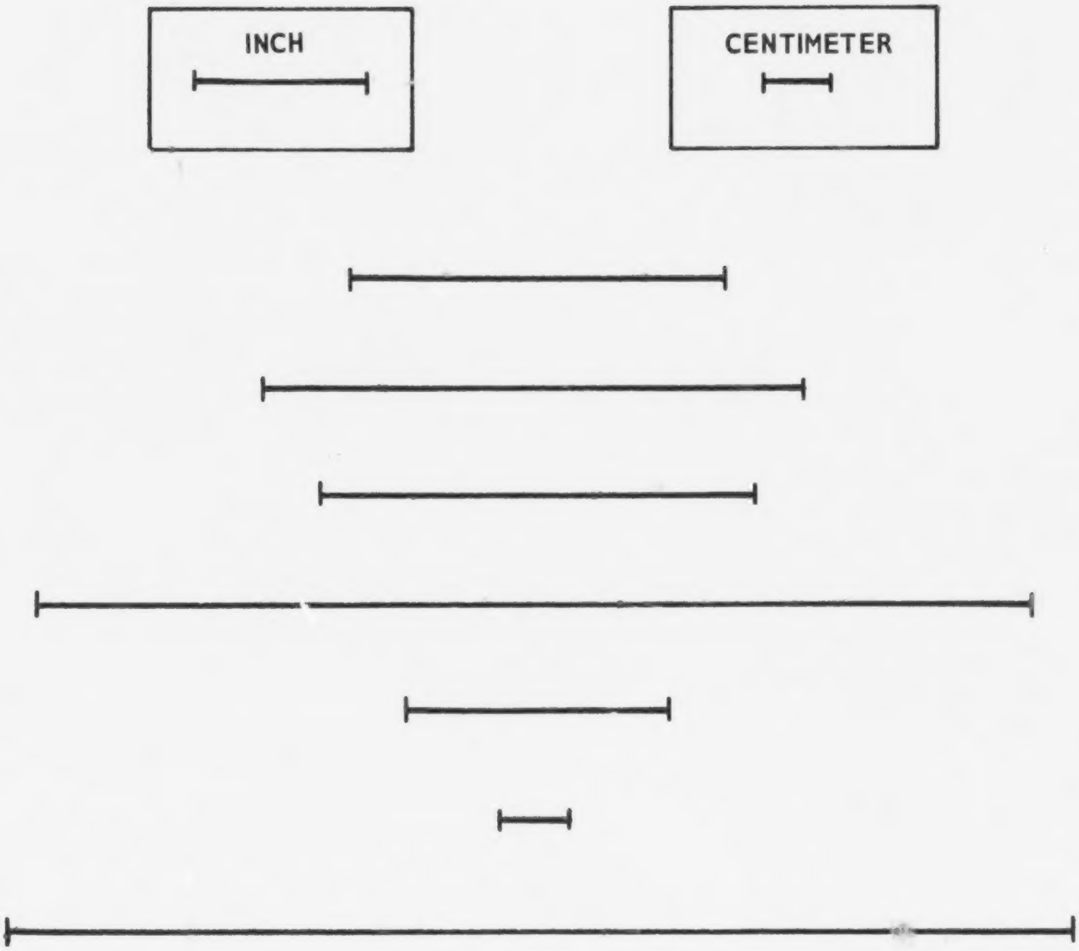
This lesson will be developed using an overhead projector and prepared transparencies. The guidelines for ruler use (p.47) and the comments on the inability to eliminate all errors in measuring (p.48), illustrating the approximate nature of measurement, should be discussed in conjunction with the transparencies. Topics from previous lessons can be reviewed by asking questions such as those included in the exercises of Lesson 7.

Comments on the use of the transparencies are listed on on pages T 49 through T 51. Note that TR 8-5, TR 8-6, and TR 8-7 are reproduced in the student booklet so that each student may attempt the measurements required on their own and have immediate feedback and discussion of their results via the use of the transparencies, acetate rulers, and an overhead projector.

Although only one objective (with four precisions) is listed for Lesson 8, this skill requires a great deal of practice and encompasses many sub-skills. For many classes, the development of this lesson and discussion of the material on the transparencies will take more than one class period.

TR 8 -1

This transparency illustrates Guideline # 5 (p.47). If you wish to discuss this guideline in detail (other than indicating the guideline in the student booklet) use this transparency in conjunction with the comments at the bottom of page T 47.


INCH
CENTIMETER

TR 8-2, 8-3, and 8-4

The competency of the class will determine how many of these examples need to be discussed. Few text developments give sufficient practice in reading rulers to tenths of an inch or tenths of a centimeter (mm.). Thus, transparency 8-4 should probably be given special attention. Use the T-Square overlay to obtain the readings to the precision indicated on each ruler. Use the length of the smallest ruler subdivision as the unit in finding the measures for the segments pictured.

A related activity would be to project the acetate rulers on the screen and ask questions similar to those in the EXERCISES of the student booklet. In particular, ask the students some questions similar to those in exercise 2, p.52.

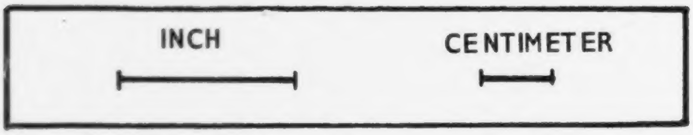
Another topic for discussion would be how the ruler in $\frac{1}{16}$ in. can be used to replace all the rulers on TR 8-2 and 8-3 except for the one graduated in tenths of an inch.

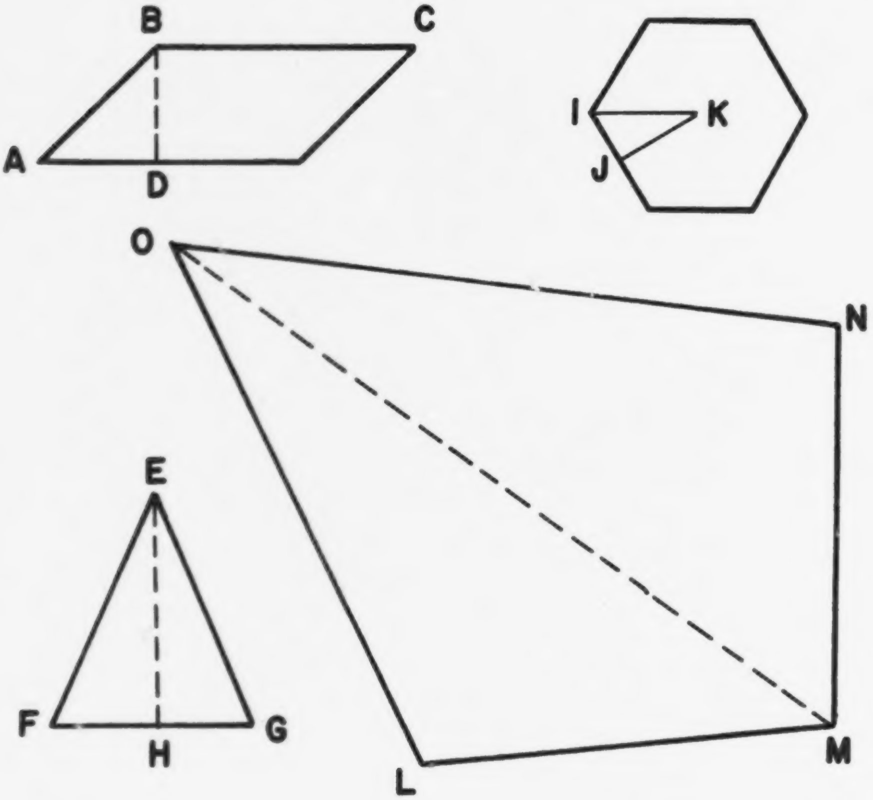
TR 8-5 and 8-6

The competency of the class will determine how much of this material needs to be re-developed, and how much is merely a review. Some classes will be able to skip parts of these two transparencies.

It is essential that the estimation sections of the problems be done. The ability to make good estimations is a practical skill and should be practiced whenever possible. Student may enjoy competing as individuals or as teams trying to obtain the "best" estimate.

The drawings on TR 8-5 and 8-6 are reproduced in the student booklet so the students can use their own rulers to measure the segments and have their work checked by demonstration on the overhead.


INCH
CENTIMETER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M

TR 8-7

More work with scale drawings wil be done later. The scale drawings here are used to motivate the student to measure.

Questions which can be used with TR 8-7 are listed below.

Questions for TR 8-7

Armadillo, Swordfish, and Bluejay

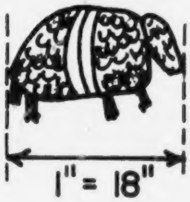
1. What is the length (height) of the scale drawing? (to the nearest $\frac{1}{8}$ inch)
2. Using the given scale, find the actual length (height).
   a) in inches b) in feet

DC-3 and SUPERSONIC TRANSPORT

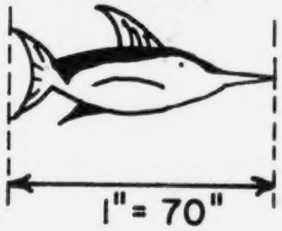
1. What is the length of the scale drawing (to the nearest $\frac{1}{4}$ inch)?
2. Using the given scale, what is the actual length of the plane in feet?

SCALE DRAWINGS

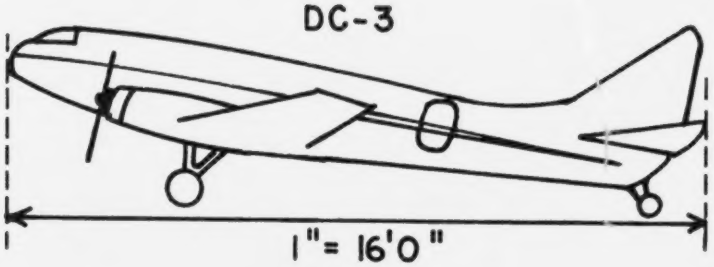
ARMADILLO

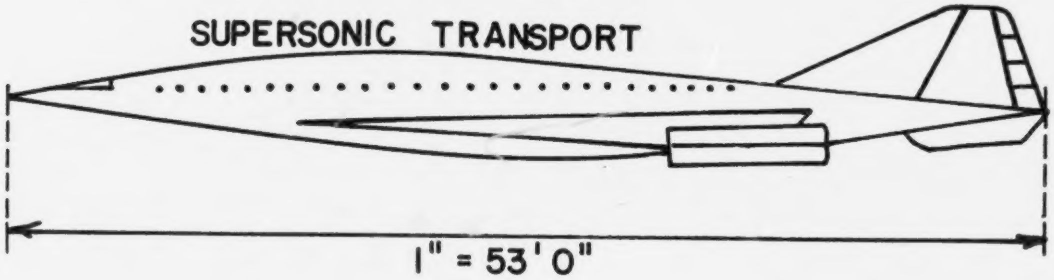


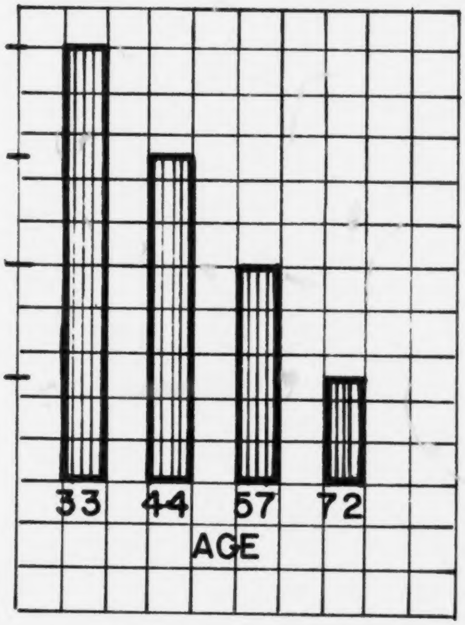
SWORDFISH



DC-3



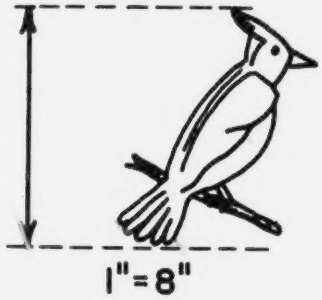

SUPERSONIC TRANSPORT
1" = 53'0"



LIFE EXPECTANCY

From Life Insurance Fact Book - 1968
Institute of Life Insurance.
Years are to the nearest whole year.

BLUE JAY



BAR GRAPH

1. The height of the bar for age 33 represents 40 years. Each unit on the vertical axis (there are four) represents how many years?

2. What is the life expectancy of a person who reaches the age of 44? of 57? of 72?

3. Using the information given what would be an estimate for the life expectancy of a person who reaches the age of 38? of 50? (The actual data is 35.74 years for a person of 38 and 25.29 for a person of 50.)

General Questions

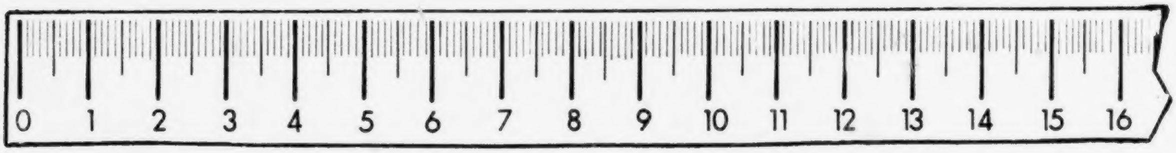
1. The actual length of the supersonic transport is about 5 times the actual length of the DC-3. Why is its drawing not 5 times the length of the DC-3? Under what conditions would its drawing be 5 times the length of DC-3 drawing?

2. Assume the same scale as given on the scale drawings. Complete the following:

Subject	Length of scale drawing	Actual length
a) Bluejay	$1\frac{1}{2}$"	?
b) Bluejay	?	10"
c) Swordfish	2"	?
d) Armadillo	$1\frac{1}{2}$"	?
e) Armadillo	?	24"

EXERCISES

1. The ruler below is graduated in tenths of a centimeter.



Indicate by a letter and arrow each of the following locations on the above ruler.

a. 6.4 cm.

b. 10.0 cm.

c. 2.2 cm.

d. 7.3 cm.

2. For each part, draw a segment which fits the given description. Draw each segment in the space below its description. A starting point for each segment is given.

a. $\overline{RS}$: $3\frac{1}{2}$ in. $<$ m($\overline{RS}$) $<$ 4 in.

To the nearest $\frac{1}{2}$ in., m($\overline{RS}$) = $3\frac{1}{2}$ in.

R.

b. $\overline{OR}$: 3.2 cm. $<$ m($\overline{OR}$) $<$ 3.3 cm.

To the nearest .1 cm., m($\overline{OR}$) = 3.3 cm.

O.

c. $\overline{WX}$: $2\frac{1}{4}$ in. $<$ m($\overline{WX}$) $<$ $2\frac{2}{4}$ in.

To the nearest $\frac{1}{4}$ in., m($\overline{WX}$) = $2\frac{1}{4}$ in.

W.

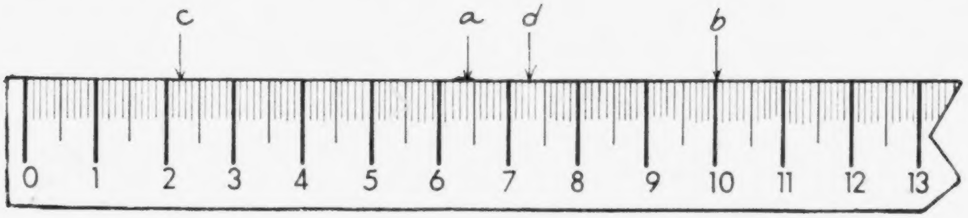
d. $\overline{GH}$: To the nearest $\frac{1}{16}$ in., m($\overline{GH}$) = $2\frac{11}{16}$ in.

G.

e. $\overline{IJ}$: To the nearest cm., m($\overline{IJ}$) = 8 cm.

.I

ANSWERS



2. Each of these must be checked individually. Properties of correct responses are listed below. An acetate overlay which can be used to check answers is provided.

a. $3\frac{1}{2}$ in. $\leq m(\overline{RS}) < 3\frac{3}{4}$ in.

b. 3.25 cm. $\leq m(\overline{OR}) < 3.3$ cm.

c. $2\frac{1}{4}$ in. $\leq m(\overline{WX}) < 2\frac{3}{8}$ in.

d. $2\frac{10\frac{1}{2}}{16}$ in. $\leq m(\overline{GH}) < 2\frac{11\frac{1}{2}}{16}$ in.

or

$2\frac{21}{32}$ in. $\leq m(\overline{GH}) < 2\frac{23}{32}$ in.

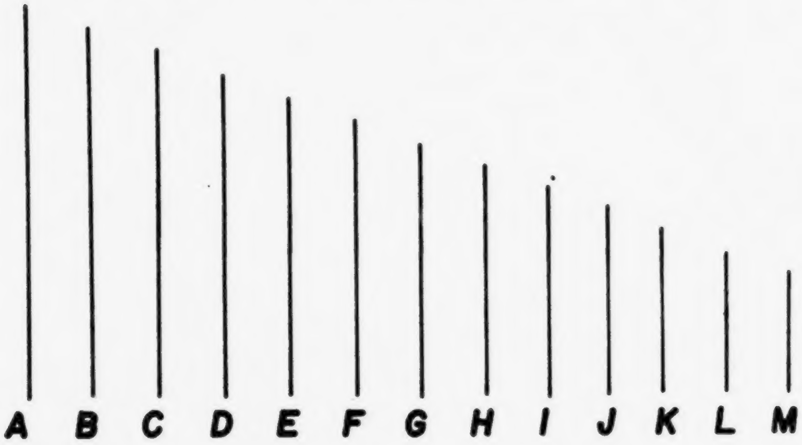
e. 7.5 cm. $< m(\overline{IJ}) < 8.5$ cm.

NOTE: There are a variety of correct responses to exercise 2. This exercise may be used to reinforce the approximate nature of measurement. A measure establishes a range for a length, it does not give an exact indication of the length. When discussing the results for d and e, ask "How long could a correct response be?" and "How short could a correct response be?"

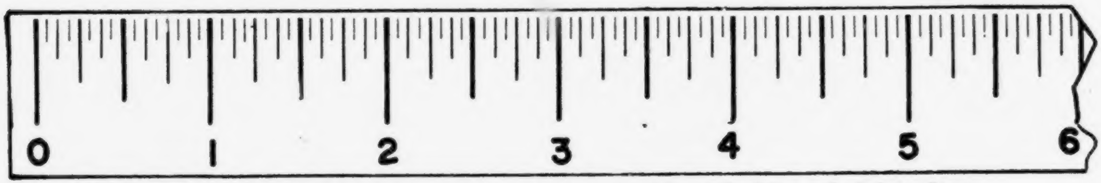
3. Estimate which segment below is nearest to...

a. two inches in length. __________

b. two centimeters in length. ________



4. The ruler below is graduated in sixteenths of an inch.



Indicate by a letter and arrow each of the following locations on the above ruler scale.

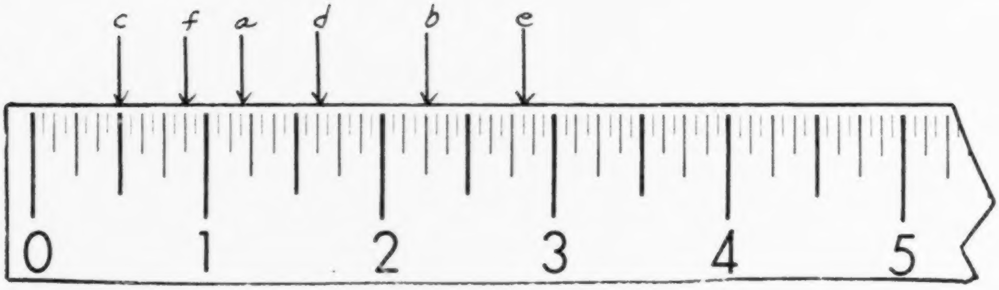
a. $1\frac{3}{16}$ in.	d. $1\frac{5}{8}$ in.
b. $2\frac{1}{4}$ in. ($2\frac{4}{16}$ in.)	e. $2\frac{13}{16}$ in.
c. $\frac{1}{2}$ in.	f. $\frac{7}{8}$ in.

ANSWERS

3. a. C

b. L

4.


c
f
a
d
b
e
0
1
2
3
4
5

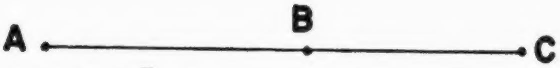
5. For each ruler location, circle the whole inch measure to which it is closest. If it is equally close to both, circle both.

	RULER LOCATION	**WHOLE INCHES**	
a.	$2\frac{3}{8}$ in.	2 in.	3 in.
b.	$\frac{9}{16}$ in.	1 in.	0 in.
c.	$1\frac{7}{16}$ in.	1 in.	2 in.
d.	$1\frac{15}{16}$ in.	3 in.	1 in.
e.	$\frac{3}{8}$ in.	0 in.	1 in.

For questions 6 and 7 do the measuring and drawing required and complete the blanks so that each statement is true. Some parts are done as examples.

6. a. Draw $\overline{AB}$ $1\frac{1}{2}$ in. long. Draw $\overline{BC}$ $1\frac{1}{4}$ long.

$m(\overline{AC}) =$ **$2\frac{3}{4}$** in. $1\frac{1}{2} + 1\frac{1}{4} =$ ______ .



b. Draw $\overline{RS}$ $\frac{7}{8}$ in. long. Draw $\overline{ST}$ $\frac{3}{16}$ in. long.

$m(\overline{RT}) =$ _____ in. $\frac{7}{8} + \frac{3}{16} =$ ______ .

c. Draw $\overline{WX}$ 2.8 cm. long. Draw $\overline{XY}$ 3.5 cm. long.

$m(\overline{WY}) =$ ______ cm. $2.8 + 3.5 =$ ______.

ANSWERS

5. a. 2 inches

b. 1 inch

c. 1 inch

d. 1 inch

e. 0 inches

These are the responses that should have been circled.

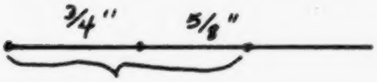
6. a. $1\frac{1}{2} + 1\frac{1}{4} = \underline{2\,\frac{3}{4}}$

b. $m(\overline{RT}) = 1\frac{1}{16}$ in. $\frac{7}{8} + \frac{3}{16} = \underline{\frac{17}{16}}$ or $1\frac{1}{16}$ in.

c. $m(\overline{WY}) = \underline{6.3}$ cm. $2.8 + 3.5 = \underline{6.3}$

NOTE: The partially worked example (6a) indicates how the segments for each part are to drawn in relation to each other.

7. Use the given line segment to compute the following sums. The precision of measurement is given in parentheses.

a.  (to nearest $\frac{1}{8}$ in.)

$\frac{3}{4} + \frac{5}{8} =$ ____

b. $\frac{7}{16} + \frac{7}{8} =$ ____

(to nearest $\frac{1}{16}$ in.)

c. $\frac{3}{16} + 1\frac{3}{4} =$ ____

(to nearest $\frac{1}{16}$ in.)

d. $5.8 + 3.9 =$ ____

(to nearest .1 cm.)

e. $2.7 + 1.6 =$ ____

(to nearest .1 in.)

f. $1\frac{5}{8} + 2\frac{7}{8} =$ ____

(to nearest $\frac{1}{8}$ in.)

ANSWERS

7. a. $\frac{3}{4} + \frac{5}{8} \quad \underline{\frac{11}{8} \text{ or } 1\,\frac{3}{8}}$.

b. $\frac{7}{16} + \frac{7}{8} = \underline{\frac{21}{16} \text{ or } 1\,\frac{5}{16}}$

c. $\frac{3}{16} + 1\,\frac{3}{4} = \underline{1\,\frac{15}{16}}$

d. $5.8 + 3.9 = \underline{9.7}$

e. $2.7 + 1.6 = \underline{4.3}$

f. $1\,\frac{5}{8} + 2\,\frac{7}{8} = 3\,\frac{12}{8} = \underline{4\,\frac{4}{8} \text{ or } 4\,\frac{1}{2}}$

CONTENT AND APPROACH

Exercises 6 and 7 are primarily measurement exercises in which the student practices his skill in using a ruler. The questions have been designed so that each measurement pictures a sum of two rational numbers (written in either decimal or fraction form). Hopefully, these exercises can be used to reinforce these computational skills. When discussing the results the student should be asked how these sums would be computed without measuring the segments. (Refer to Lesson 7—exercises 4 and 5—and the work on equivalent fractions in both Lesson 7 and the booklet **ACTIVITIES WITH RATIO AND PROPORTION**.)

NOTE: The partially worked example (7a) indicates how each problem is to be worked. The measurement to be done is indicated by the brace.

8. For each segment named in TABLE 8-1, (1) estimate its length, and (2) measure its length to the nearest $\frac{1}{8}$ in., $\frac{1}{16}$ in., and .1 cm.

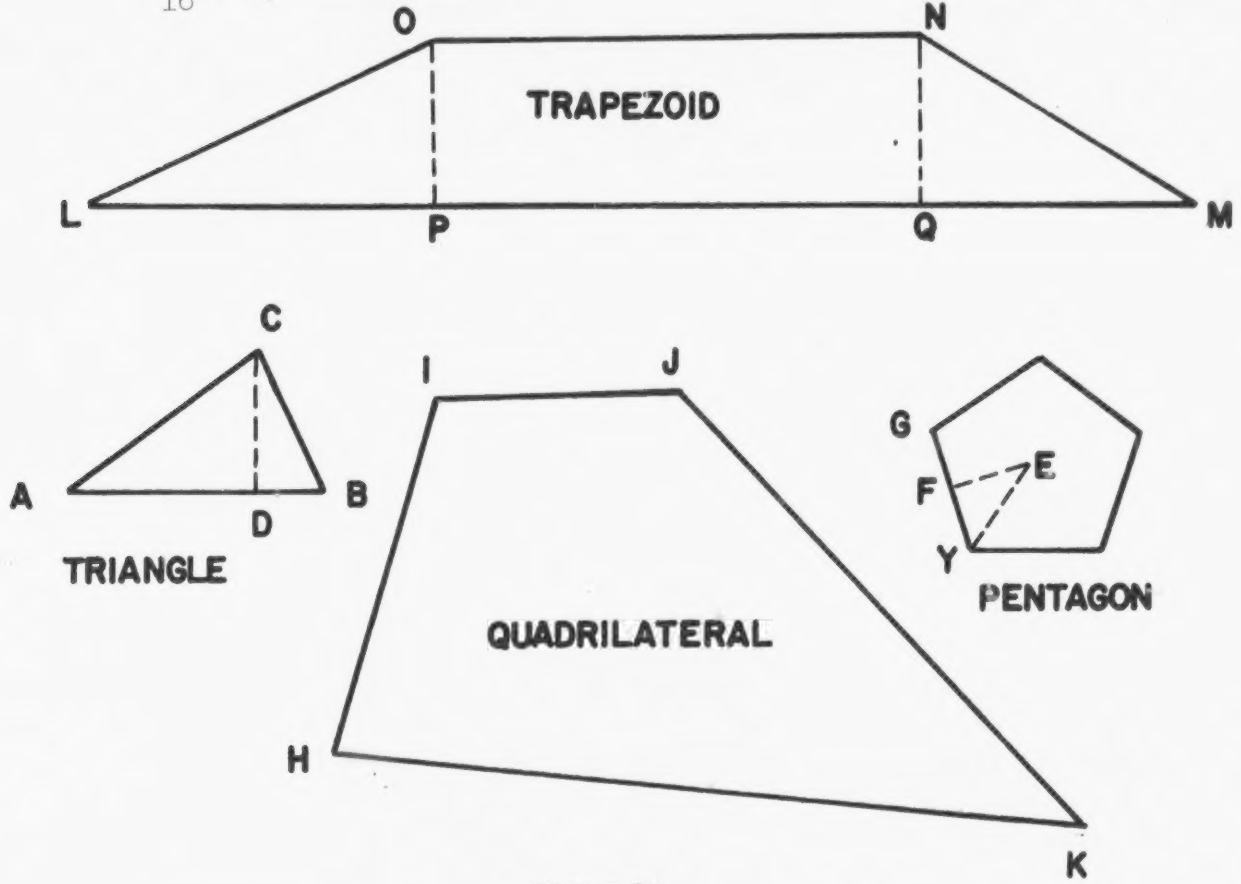

O
N
TRAPEZOID
L
P
Q
M
C
A
D
B
TRIANGLE
I
J
H
K
QUADRILATERAL
G
E
F
Y
PENTAGON

TABLE 8-1

SEGMENT	ESTIMATE	LENGTH TO THE NEAREST...		
		$\frac{1}{8}$ in.	$\frac{1}{16}$ in.	.1 cm.
$\overline{CD}$				
$\overline{OP}$				
$\overline{NQ}$				
$\overline{HK}$				
$\overline{FE}$				
$\overline{EY}$				

THINGS TO DISCUSS

Some variation in readings for exercise 5 may occur due to scale distortions on the cardboard rulers and/or inability to locate the endpoints of the segments (the line segments are drawn with a broad pen). Reasons for variation in readings could be a topic for class discussion when discussing the results.

ANSWERS

TABLE 8-1

		LENGTH TO THE NEAREST...		
SEGMENT	ESTIMATE	$\frac{1}{8}$ in.	$\frac{1}{16}$ in.	.1 cm.
$\overline{CD}$	*	$\frac{6}{8}$ in.	$\frac{13}{16}$ in.	2.0 cm.
$\overline{OP}$	*	1 in.	$\frac{15}{16}$ in. in.	2.4 cm.
$\overline{NQ}$	*	1 in.	$\frac{15}{16}$ in.	2.4 cm.
$\overline{HK}$	*	$4 \frac{3}{8}$ in.	$4 \frac{6}{16}$ in.	11.2 cm.
$\overline{FE}$	*	$\frac{4}{8}$ in.	$\frac{7}{16}$ in.	1. 1 cm.
$\overline{EY}$	*	$\frac{5}{8}$ in.	$\frac{9}{16}$ in.	1. 5 cm.

* Estimates will vary.

9. To the nearest .1 inch, the length of $\overline{HI}$ is 5.4 cm. Estimate in tenths of a centimeter the lengths of the following segments.

a. $\overline{IJ}$ ______

c. $\overline{LM}$ ______

b. $\overline{AC}$ ______

10. Find the perimeter (distance around) to the nearest .1 cm. for the quadrilateral pictured on page 56. ______

ANSWERS

9. This question reviews estimation by comparing the length to be estimated with a segment of known length(which is used as the estimation guide). The length of the segments to the nearest .1 cm. are:

   a. $m(\overline{IJ}) = \underline{3.6}$ cm. c. $m(\overline{LM}) = \underline{16.4}$ cm.
   b. $m(\overline{AC}) = \underline{3.5}$ cm.

*10. 28.9 cm.

THINGS TO DISCUSS

The following could be used in conjunction with question # 8 of the **EXERCISES**.

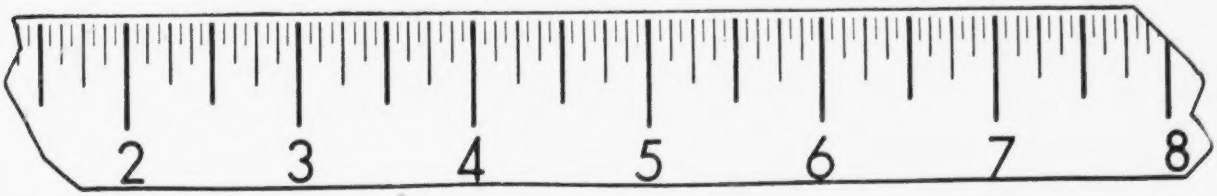
1. To the nearest .1 inch, what is the length of the altitude of △ ABC ?
2. What is the relationship between $\overline{OP}$ and $\overline{NQ}$?
   (They are the same length and parallel.)

* Inability to get accurate readings for the four segments may cause some variations in the sum (See **THINGS TO DISCUSS**, p.56). The measurements used to obtain the sum of 28.5 cm. were...

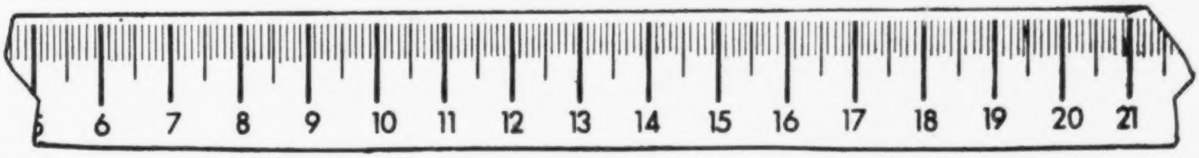
$m(\overline{IJ}) = 3.6$ cm. $m(\overline{HK}) = 11.2$ cm.
$m(\overline{JK}) = 8.7$ cm. $m(\overline{HI}) = 5.4$ cm.

WHEN RULES ARE BROKEN

In the next set of EXERCISES you will use two broken rulers like those pictured below.



RULER A


6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21

RULER B

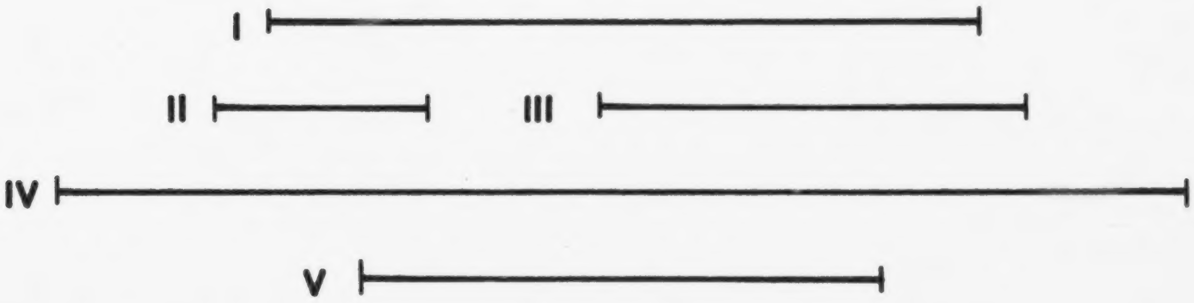
EXERCISES

1. Measure each of the following segments to the nearest...

   a. $\frac{1}{8}$ inch using ruler A, and

   b. $\frac{1}{10}$ centimeter using ruler B.

   Record your results in the TABLE 9-1. (in rows labeled Broken ruler)



WHEN RULES ARE BROKEN

OBJECTIVES

1. The student shall be able to measure accurately to the nearest $\frac{1}{8}$ in. and .1 cm., using as the origin a point other than the zero point on the ruler scale.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

*1. Rulers graduated in (a) $\frac{1}{16}$ in. (b) .1 cm.

*2. Broken rulers graduated in (a) $\frac{1}{16}$ in. (b) .1 cm.

B. TEACHER

*1. Acetate rulers (graduated like those provided the student).

2. Overhead projector

3. Projector screen

CONTENT AND APPROACH

The student is required to measure the five segments on page 42 using both the broken rulers and rulers used in previous lessons.

If you feel the additional practice would be valuable, the student could be asked to estimate the lengths for some of the segments prior to measuring.

Notice that the ruler used to measure to the nearest $\frac{1}{8}$ inch is graduated in sixteenths of an inch. It may be necessary to review with the class the meaning of the various markings (which markings represent $\frac{1}{2}$ inches, $\frac{1}{4}$ inches,...) prior to **EXERCISES II** . An acetate ruler can be used with an overhead projector to project a picture of the ruler during the discussion.

TABLE 9-1

SEGMENT	LENGTH TO THE NEAREST...		
	$\frac{1}{8}$ inch	$\frac{1}{10}$ cm.	
I			Broken ruler
			Unbroken ruler
II			Broken ruler
			Unbroken ruler
III			Broken ruler
			Unbroken ruler
IV			Broken ruler
			Unbroken ruler
V			Broken ruler
			Unbroken ruler

2. Measure each of the same segments to the same precisions using rulers that are not broken. Record your results in TABLE 9-1. (in rows labeled Unbroken ruler)

DISCUSSION QUESTIONS

1. For problem 1, did everyone in the class use the same point on the ruler as their "zero point"?

2 Could you use any point on the ruler scale as the "zero point"? Would some points be more convenient to use than others?

3. Why were the measures using either the broken ruler or the unbroken ruler the same?

START WHERE YOU WANT TO...

The previous EXERCISES demonstrated that it is not necessary to use the point labeled zero as the "zero point" when reading a ruler. In fact, there are times when it may be preferable to begin your reading with some point other than the "zero point" labeled on the ruler.

ANSWERS

TABLE 9-1

LENGTH TO THE NEAREST...

SEGMENT	$\frac{1}{8}$ inch	$\frac{1}{10}$ cm.
I	$4\frac{1}{8}$	10.6 cm.
II	$1\frac{2}{8}$ in.	3.1 cm.
III	$2\frac{4}{8}$ in.	6.3 cm.
IV	$6\frac{5}{8}$ in.	16.9 cm.
V	3 in.	7.7 cm.

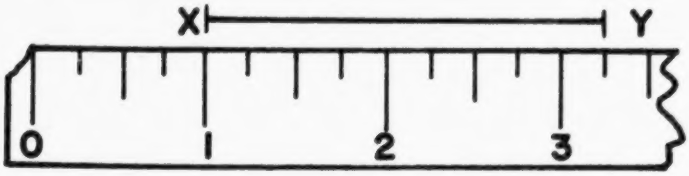
NOTE: The measures using the broken and unbroken rulers are the same for each segment.

The **DISCUSSION QUESTIONS** on page 59 discuss the results of **TABLE 9-1**. The major conclusion of this section is brought out through the discussion of question 2. The answer to the first part of this question is obviously "yes". Some points on the ruler scale (like $\frac{1}{4}$, $\frac{1}{2}$, 1,...) are more convenient to use because they simplify the computation and counting necessary to determine the measure when using a point other than zero as the origin.

START WHERE YOU WANT TO discusses situations where the use of a point other than the zero point of the ruler is preferable. It is important that the students see a possible need for the measuring done in this lesson. This section can be used as an extension of the **DISCUSSION QUESTIONS** on page 59.

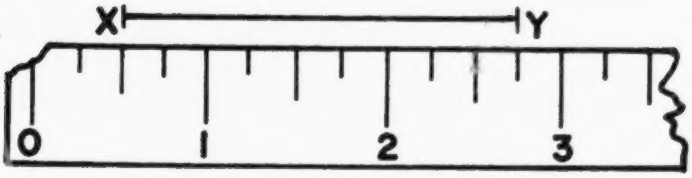
For example, the ruler pictured below has an end which is "broken off" making the zero point hard to locate. Therefore, it would be preferable to use another point as the starting point. Shown below is $\overline{XY}$ being measured using three different points as the "zero point."

A.



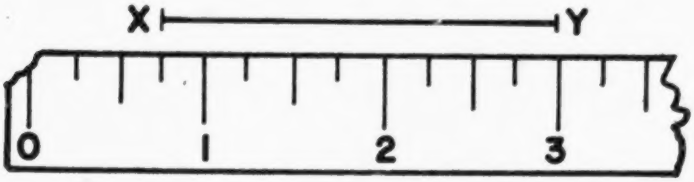
$m(\overline{XY}) = 3\frac{1}{4} - 1 = 2\frac{1}{4}$ inches

B.



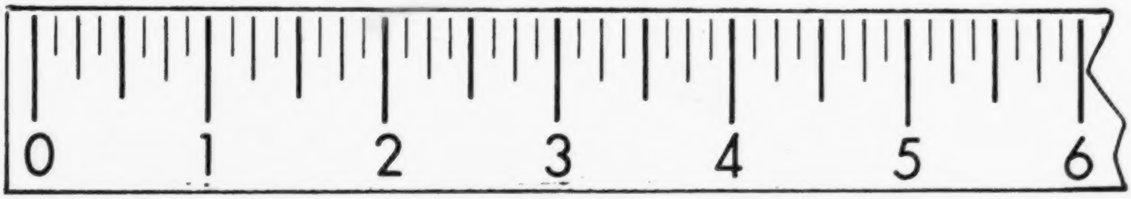
$m(\overline{XY}) = 2\frac{3}{4} - \frac{2}{4} = 2\frac{1}{4}$ inches

C.



$m(\overline{XY}) = 3 - \frac{3}{4} = 2\frac{1}{4}$ inches

In all cases, $m(\overline{XY}) = 2\frac{1}{4}$ inches when measured to the nearest $\frac{1}{4}$ inch. Because ruler edges tend to wear and become rounded with use, some rulers inset the zero point of the scale so that the zero point can be determined despite the rounding. (See sketch below.) The rulers you are using in this unit are marked in this manner.



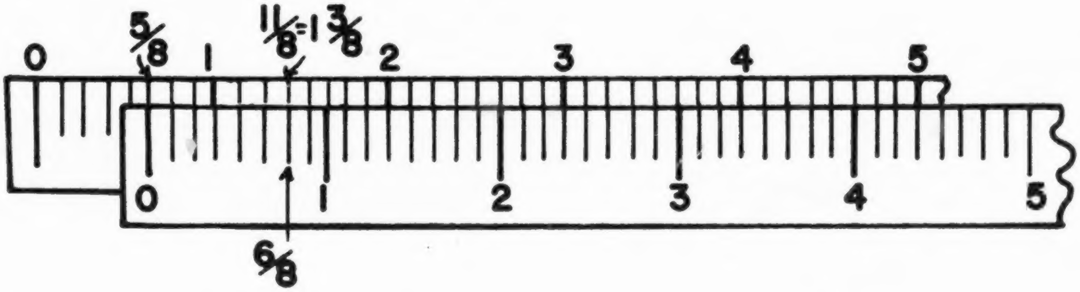
THINGS TO DISCUSS

Notice the comment at the bottom of page 60 concerning the inset zero point (see page T 41 for additional comments). In most instances the inset zero point is preferable. However, there are situations in which it would be preferable to have the zero point at the end of the ruler. For example, if you were measuring the height of a picture from the floor, this height could be read directly from the ruler scale if the zero point is at the end of the ruler.

ADDITION AND SUBTRACTION WITH RULERS

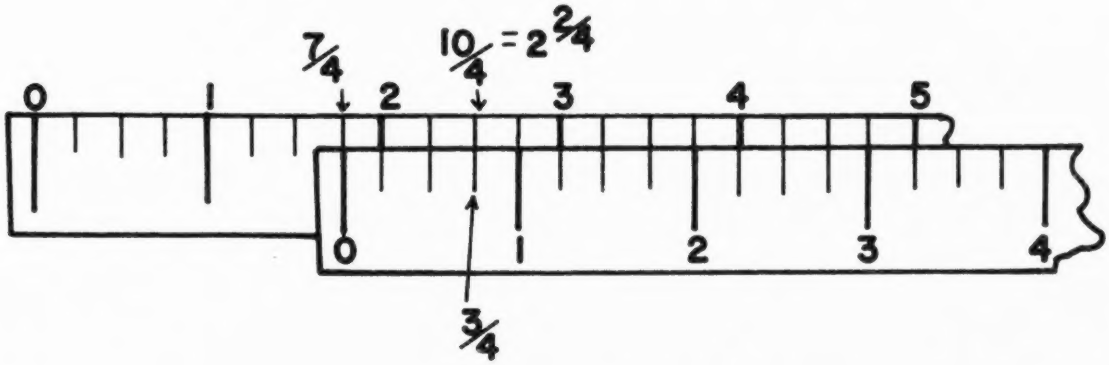
Lessons 7 and 8 included some exercises in which rulers and segment lengths were used to illustrate the addition and equivalence of certain fractions.

For example, the rulers below are illustrating the sum $\frac{5}{8} + \frac{6}{8} = \frac{11}{8} = 1\frac{3}{8}$.



The same picture can be used to illustrate two related subtraction facts:

$$1\frac{3}{8} - \frac{5}{8} = \frac{6}{8} \quad \text{and} \quad 1\frac{3}{8} - \frac{6}{8} = \frac{5}{8}.$$



The two rulers above are illustrating the sum $\frac{7}{4} + \frac{3}{4} = \frac{10}{4} = 2\frac{2}{4}$ (This could be written $1\frac{3}{4} + \frac{3}{4} = 2\frac{2}{4}$).

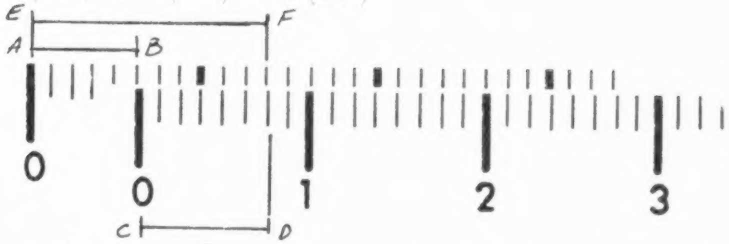
What two subtraction facts are related to this sum?

CONTENT AND APPROACH

As in Lessons 7 and 8, the exercises are primarily measurement activities. However, rulers can be used to illustrate some properties of and operations with rational numbers. When some point of a ruler scale other than zero is used as the "zero point" it is sometimes necessary to subtract rational numbers (the readings at each of the end-points) to obtain the measure of a segment. For this reason, some practice with subtraction of rational numbers is included in Lesson 9.

On the first ruler, to illustrate the sum $\frac{5}{8} + \frac{6}{8}$ the following lengths are combined (see picture below):

$$m(\overline{AB}) + m(\overline{CD}) = m(\overline{EF})$$



$1\frac{3}{8} - \frac{5}{8} = \frac{6}{8}$ is represented by $m(\overline{EF})-m(\overline{AB})=m(\overline{CD})$

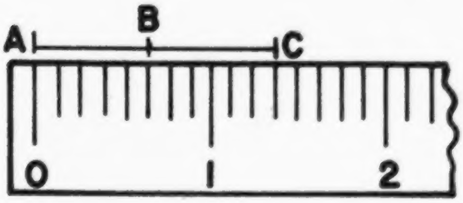
$1\frac{3}{8} - \frac{6}{8} = \frac{5}{8}$ is represented by $m(\overline{EF})-m(\overline{CD})=m(\overline{AB})$

The two subtraction facts related to the sum $\frac{7}{4} + \frac{3}{4} = 2\frac{2}{4}$ are $2\frac{2}{4} - \frac{7}{4} = \frac{3}{4}$ and $2\frac{2}{4} - \frac{3}{4} = \frac{7}{4}$.

When discussing these illustrations, computational skills can be reviewed by asking how these sums and differences can be computed without using the rulers. (See page T 62 for strategies.)

Segment lengths could also be used to illustrate the addition and subtraction of fractions.

For example, consider the sum $\frac{5}{8} + \frac{6}{8}$.



$m(\overline{AC}) = m(\overline{AB}) + m(\overline{BC})$

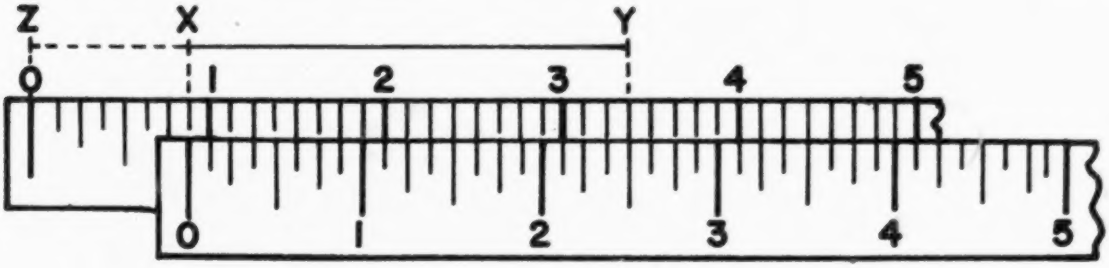
$m(\overline{AC}) = \frac{5}{8}$ in. $+ \frac{6}{8}$ in.

$m(\overline{AC}) = \frac{11}{8}$ in. $= 1\frac{3}{8}$ in.

$$\frac{5}{8} + \frac{6}{8} = \frac{11}{8} = 1\frac{3}{8}$$

Since $m(\overline{AC}) - m(\overline{AB}) = m(\overline{BC})$ and $m(\overline{AC}) - m(\overline{BC}) = m(\overline{AB})$, the picture also illustrates $1\frac{3}{8} - \frac{5}{8} = \frac{6}{8}$ and $1\frac{3}{8} - \frac{6}{8} = \frac{5}{8}$.

What is $m(\overline{XY})$ in the figure below?



$m(\overline{XY}) = m(\overline{ZY}) - m(\overline{ZX})$

$m(\overline{XY}) = 3\frac{3}{8}$ in. $- \frac{7}{8}$ in.

$m(\overline{XY}) = \frac{27}{8}$ in. $- \frac{7}{8}$ in.

But the bottom ruler shows that $m(\overline{XY}) = 2\frac{4}{8}$ or $\frac{20}{8}$ in.

Therefore, $3\frac{3}{8} - \frac{7}{8} = \frac{27}{8} - \frac{7}{8} = \frac{20}{8} = 2\frac{4}{8}$.

Some strategies which can be used (refer to work on equivalent fractions previously done in this booklet and ACTIVITIES WITH RATIO AND PROPORTION) in reviewing the computational skills illustrated on pages 61-63 are listed below.

I. a. $1\frac{3}{8} - \frac{5}{8} = (1 + \frac{3}{8}) - \frac{5}{8} = (\frac{8}{8} + \frac{3}{8}) - \frac{5}{8} = \frac{11}{8} - \frac{5}{8} = \frac{6}{8}$

b. $1\frac{3}{8} - \frac{5}{8} = (1 + \frac{3}{8}) - \frac{5}{8} = (\frac{8}{8} + \frac{3}{8}) - \frac{5}{8} = (\frac{8}{8} - \frac{5}{8}) + \frac{3}{8} =$

$\frac{3}{8} + \frac{3}{8} = \frac{6}{8}$

II. a. $2\frac{2}{4} - 1\frac{3}{4} = (2 + \frac{2}{4}) - (1 + \frac{3}{4}) = (\frac{8}{4} + \frac{2}{4}) - (\frac{4}{4} + \frac{3}{4}) =$

$\frac{10}{4} - \frac{7}{4} = \frac{3}{4}$

b.

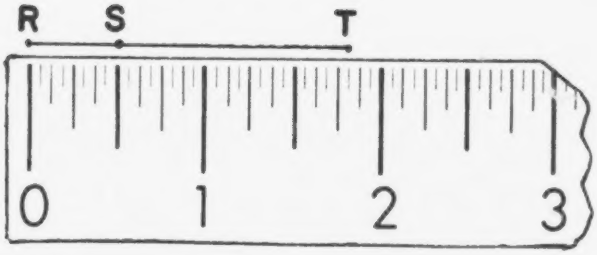
$$\begin{array}{cccc} 2\frac{2}{4} & (1 + 1) + \frac{2}{4} & 1 + (1 + \frac{2}{4}) & 1 + \frac{6}{4} \\ \underline{-\ 1\frac{3}{4}} & \underline{-\ \ 1 + \frac{3}{4}} & \underline{-\ 1 + \frac{3}{4}} & \underline{-\ 1 + \frac{3}{4}} \\ & & & 0 + \frac{3}{4} = \frac{3}{4} \end{array}$$

c. $2\frac{2}{4} - 1\frac{3}{4} = (2 + \frac{2}{4}) - (1 + \frac{3}{4}) = (1 + 1 + \frac{2}{4}) - (1 + \frac{3}{4}) =$

$(1 + \frac{6}{4}) - (1 + \frac{3}{4}) = (1 - 1) + (\frac{6}{4} - \frac{3}{4}) = \frac{3}{4}$

Additional practice with equivalent fractions can be given by asking the student to write sums and differences in simplest form. Notice that although $\frac{6}{8}$ in. and $\frac{3}{4}$ in. represent the same point on a ruler scale, $\frac{6}{8}$ in. and $\frac{3}{4}$ in. do not represent the same measure as $\frac{6}{8}$ in. represents a measurement to the nearest $\frac{1}{8}$ in. and $\frac{3}{4}$ in. represents a measurement to the nearest $\frac{1}{4}$ in. This is not a critical distinction at this stage but will become critical when precision is discussed in more detail and measurements will be written to indicate the precision of the measurement.

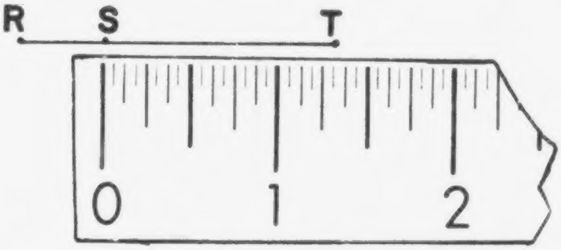
As no use is made of this distinction in this booklet, There is probably need to discuss it at this time. However, avoid giving the students the impression that $\frac{6}{8}$ in. = $\frac{3}{4}$ in. as this may create problems later on.



m($\overline{RT}$) = $1\frac{13}{16}$ inches.

m($\overline{RS}$) = $\frac{1}{2}$ inch.

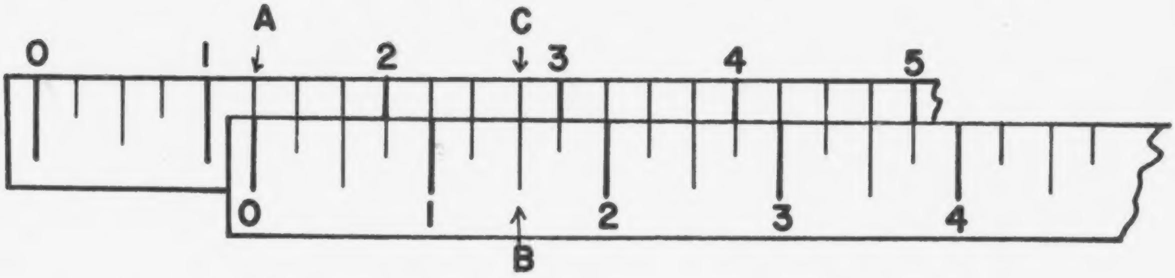
m($\overline{ST}$) = ___?___ in.



m($\overline{ST}$) = m($\overline{RT}$) - m($\overline{RS}$)
m($\overline{ST}$) = $1\frac{13}{16}$ in. - $\frac{1}{2}$ in.
m($\overline{ST}$) = $1\frac{5}{16}$ in.

Therefore, $1\frac{13}{16} - \frac{1}{2} = 1\frac{13}{16} - \frac{8}{16} = 1\frac{5}{16}$

✓ **POINT**



1. Give the sum illustrated by the two rulers above.

   (A + B = C) __________

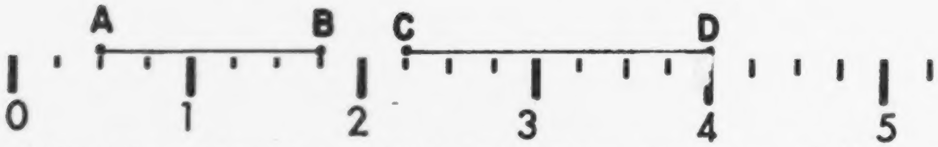
2. Give the two subtraction facts related to the above sum.

ANSWERS

1. $1\frac{1}{4} + 1\frac{2}{4} = 2\frac{3}{4}$

2. $2\frac{3}{4} - 1\frac{1}{4} = 1\frac{2}{4}$ and $2\frac{3}{4} - 1\frac{2}{4} = 1\frac{1}{4}$

NOTE: Allow for variations in the manner in which the fractions are expressed. For example, the answer to # 1 could be written as $\frac{5}{4} + \frac{6}{4} = \frac{11}{4}$ or $1\frac{1}{4} + 1\frac{1}{2} = 2\frac{3}{4}$.


A
B
C
D
0
1
2
3
4
5

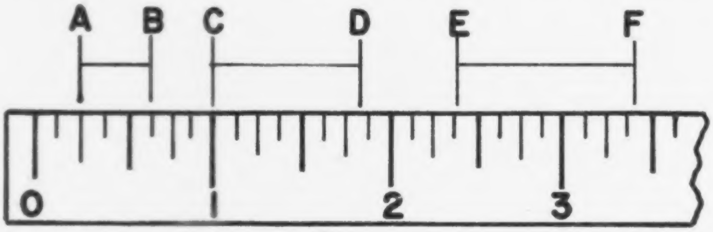
3. Give the measures of the following segments to the nearest $\frac{1}{4}$ inch.

a. $m(\overline{AB})$ = ____ - ____ = ____ in.

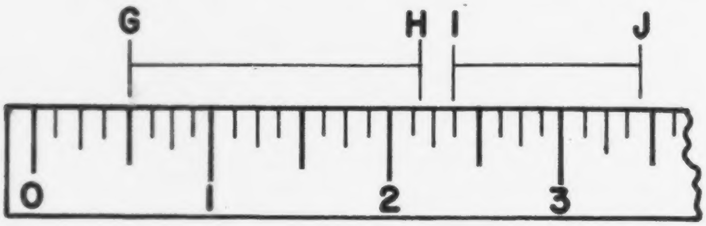
b. $m(\overline{CD})$ = _____ in.

EXERCISES

1. Find the length of each segment below to the nearest $\frac{1}{8}$ inch.

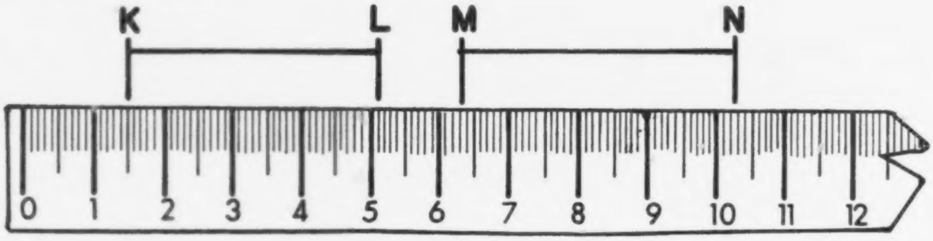

A
B
C
D
E
F
0
1
2
3

$m(\overline{AB})$ = _________ $m(\overline{CD})$ = _________ $m(\overline{EF})$ = _______



$m(\overline{GH})$ = ________ $m(\overline{IJ})$ = __________

2. Find the length of the segments below to the nearest .1 centimeter.


K
L
M
N
0
1
2
3
4
5
6
7
8
9
10
11
12

$m(\overline{KL})$ = _________ $m(\overline{MN})$ = ____________

ANSWERS

✓ POINT

3. a. $m(\overline{AB}) = \underline{1\frac{3}{4}} - \underline{\frac{2}{4}} = \underline{1\frac{1}{4}}$ in.

   b. $m(\overline{CD}) = \underline{1\frac{3}{4}}$ in.

EXERCISES II

1. $m(\overline{AB}) = \frac{3}{8}''$ $m(\overline{CD}) = \frac{7}{8}''$ $m(\overline{EF}) = 1''$

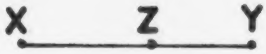
   $m(\overline{GH}) = 1\frac{5}{8}''$ $m(\overline{IJ}) = 1\frac{1}{8}''$ (This one is close - some may answer 1".)

2. $m(\overline{KL}) = 3.6$ cm. $m(\overline{MN}) = 4.0$ cm.

3. In question 3, do the measuring and drawing required and complete the blanks so that each statement is true. One part is done as an example.

a. Draw $\overline{XY}$ $1\frac{5}{16}$ in. long. Draw $\overline{XZ}$ $\frac{3}{4}$ in. long.

$m(\overline{ZY}) =$ 9/16 in $1\frac{5}{16} - \frac{3}{4} =$ ______



b. Draw $\overline{PR}$ $2\frac{3}{4}$ in. long. Draw $\overline{PS}$ $1\frac{3}{8}$ in. long.

$m(\overline{SR}) =$ _____ in. $2\frac{3}{4} - 1\frac{1}{8} =$ ______

c. Draw $\overline{OD}$ 6.8 cm. long. Draw $\overline{OR}$ 3.5 cm. long.

$m(\overline{RD}) =$ _____ cm. 6.8 - 3 5 = ______

d. Draw $\overline{AS}$ $3\frac{7}{8}$ in. long. Draw $\overline{AT}$ $1\frac{3}{16}$ in. long.

$m(\overline{TS}) =$ _____ in. $3\frac{7}{8} - 1\frac{3}{16} =$ ______

4. Compute the following differences.

a. $\frac{13}{16} - \frac{3}{8} =$ _____

b. $2\frac{1}{2} - 1\frac{3}{8} =$ _____

c. 6.2 - 4.5 = _____

d. $\frac{7}{8} - \frac{3}{4} =$ ______

ANSWERS

3. a. Done as an example in the student text.

b. $m(\overline{SR}) = \underline{1\frac{3}{8}}$ in. $\qquad 2\frac{3}{4} - 1\frac{3}{8} = \underline{1\frac{3}{8}}$

c. $m(\overline{RD}) = \underline{3.3}$ in. $\qquad 6.8 - 3.5 = \underline{3.3}$

d. $m(\overline{TS}) = \underline{2\frac{11}{16}}$ in. $\qquad 3\frac{7}{8} - 1\frac{3}{16} = \underline{2\frac{11}{16}}$

4. a. $\frac{13}{16} - \frac{3}{8} = \underline{\frac{7}{16}}$

b. $2\frac{1}{2} - 1\frac{3}{8} = \underline{1\frac{1}{8}}$

c. $6.2 - 4.5 = \underline{1.7}$

d. $\frac{7}{8} - \frac{3}{4} = \underline{\frac{1}{8}}$

NOTE: Refer to pages T 61 and T 62 for comments on reviewing computational skills. It is intended that rulers will not be used to work exercise 4. However, if necessary, allow students to use them.

The partially worked example (3a) indicates how the segments for each part are to be drawn in relation to each other. The last part of 3b should read "$2\frac{3}{4} - 1\frac{3}{8}$".

5. In the space below, draw a rectangle whose longest side is $3\frac{5}{16}$ inches long and whose shortest side is $1\frac{7}{8}$ inches long.

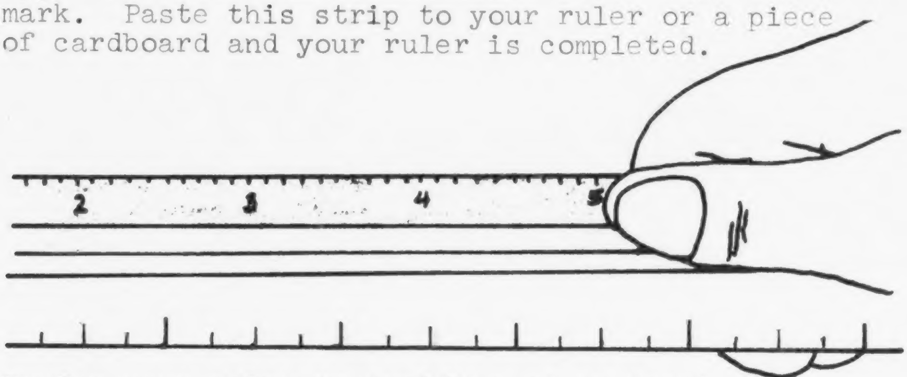
6. What is the perimeter (distance around) for the rectangle you drew in exercise 5? _____

MARKING OFF TENTHS OF AN INCH . . .

You can calibrate a ruler in <u>tenths of an inch</u> with a <u>pica typewriter</u>. Just type a series of single quotation marks on a strip of paper. The distance between each quotation mark is .1 inch.

```
''''''''''''''''''''''''''''''''''''''''''''
0         1         2         3         4
```

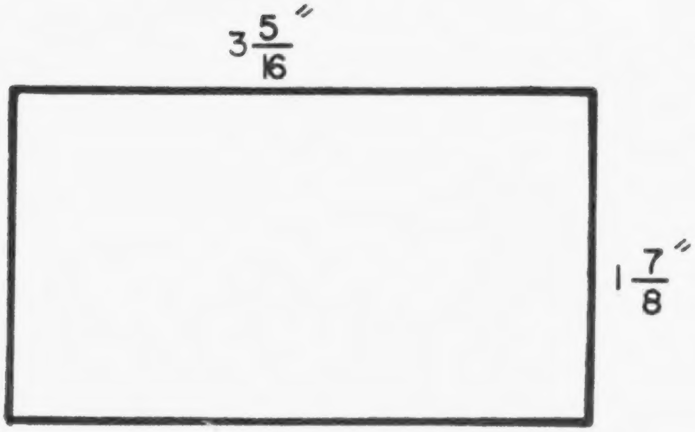
Type the whole numbers in order under each tenth mark. Paste this strip to your ruler or a piece of cardboard and your ruler is completed.



Prefer your ruler in twelfths of an inch? Use an elite typewriter.

ANSWERS

5.



6. $10\frac{3}{8}$ in.

THINGS TO DISCUSS

The article on calibrating rulers with a typewriter may be of interest to students who are taking typing. Variations of this activity using the carriage return and either the dash or underline key on a standard pica typewriter can be used to calibrate a ruler in sixths of an inch. Other precisions can be calibrated using every second, third,... mark or by changing the spacing.

✓POINT

Use the following statements and problems to check your understanding of Lessons 7-9. If you have difficulty with any questions, it may indicate a topic you should review.

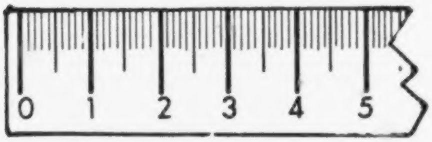
1. Supply the missing numbers so that the fractions in each part are equivalent.

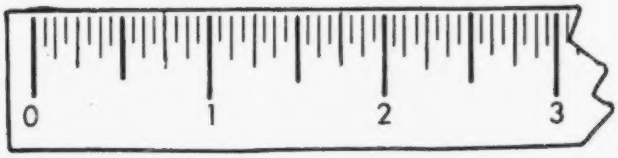
   a. If $\frac{3}{4} = \frac{x}{16} = \frac{6}{y}$, then x = ___ and y = ____.

   b. If $\frac{3}{8} = \frac{d}{16}$, then d = ________________

   c. If $\frac{1}{2} = \frac{2}{r} = \frac{s}{8}$, then r = ________ and s = _________.

   d. If $\frac{1}{2} = \frac{a}{10}$, then a = ______.





2. Indicate by a letter and arrow each of the following locations on the rulers above.

   a. 1.8 cm.
   b. $1\frac{3}{16}$ in.
   c. $2\frac{5}{8}$ in.
   d. $\frac{3}{4}$ in.
   e. $2\frac{1}{2}$ cm.
   f. $\frac{11}{16}$ in.

3. a. $3\frac{11}{16} - 1\frac{1}{4} =$ ______
   b. 9.3-2.9 = ________
   c. $\frac{7}{8} + \frac{1}{2} =$ ______
   d. $2\frac{3}{8} + 1\frac{5}{16} =$ ______

4. You should be able to measure to the nearest...

   a. $\frac{1}{2}$ in.
   b. $\frac{1}{4}$ in.
   c. $\frac{1}{8}$ in.
   d. $\frac{1}{16}$ in.
   e. .1 in.
   f. .1 cm.

   ...using any point of the ruler scale as the starting point.

ANSWERS

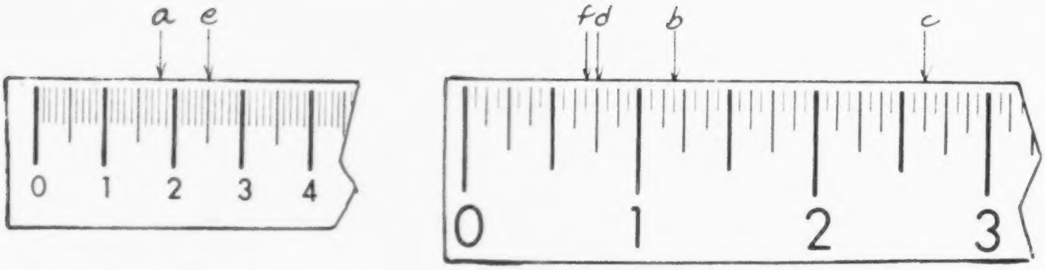
1. a. x = 12 and y = 8

   b. d = 6

   c. r = 4 and s = 4

   d. a = 5

2.



3. a. $3\frac{11}{16} - 1\frac{1}{4} = 2\frac{7}{16}$ c. $\frac{7}{8} + \frac{1}{2} = \frac{11}{8}$ or $1\frac{3}{8}$

   b. 9.3 - 2.9 = 6.4 d. $2\frac{3}{8} + 1\frac{5}{16} = 3\frac{11}{16}$

4. No response is intended. These statements are to be used as a self-check by the student.

HOW UNITS ARE RELATED

The length of an average classroom is...

a. 10 yards
b. 30 feet
c. 360 inches.

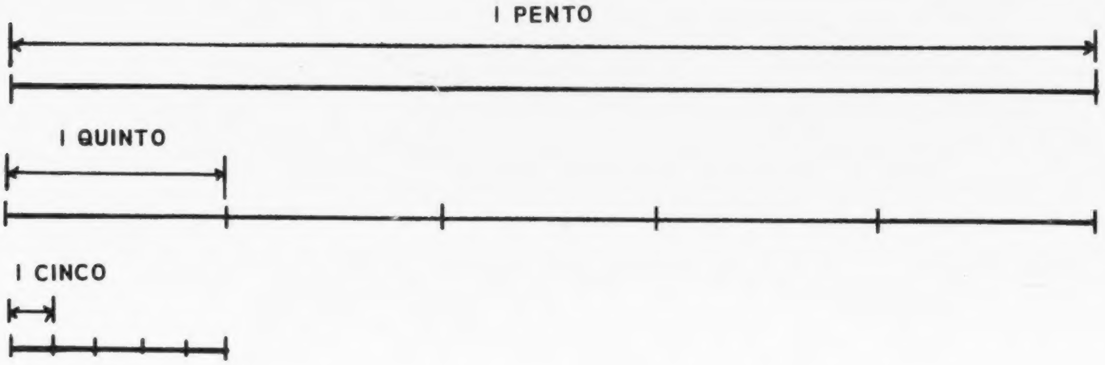
If measured to the nearest inch, the above choices are equivalent lengths. Therefore, they could be used interchangeably to complete the statement correctly.

However, in some situations one of these units may be more convenient or suitable to use than the other two. For example, the length of a room is more commonly expressed in feet or some combination of feet and inches than in inches or yards. It is sometimes necessary to convert a length given in one unit to an equal length in some other unit.

Suppose we used the following units for measuring length. At present, we will not be concerned with the lengths of these units, only the relationship between them. (shown by the following drawing)

1 pento = 5 quintos

1 quinto = 5 cincos



HOW UNITS ARE RELATED

OBJECTIVES

1. Given information such as 1 ft. = 12 in. and 4 ft. = ? in., The student shall be able to...
   a. determine whether multiplication or division should be used to find the missing value.
   b. find the missing value.
2. Given a relationship between two units such as 6 jogs = 3 bogs, the student shall be able to determine which of the two units is larger.

CONTENT AND APPROACH

This is the initial lesson of a two-lesson sequence on conversion of units. The central understandings related to the objectives of this lesson are:

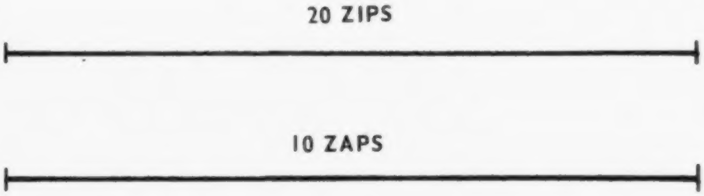
1. Given two units, it will take fewer (more) of the larger (smaller) unit to express the measure. (This idea was discussed earlier on page 10, Lesson 1).
2. The units are related linearly and a change in one unit produces a proportional change in the other unit. Without this assumption, we could not use the ratio-proportion method of conversion discussed in Lesson 11.

The lesson is designed for the student to discover the above generalizations, although he should not be expected to be able to fully verbalize them.

An optional activity would be to have the students estimate distances in pentos, quintos, and cincos. The estimates could be checked by measuring, constructing a ruler graduated in these units or using the inch equivalents (measured to nearest $\frac{1}{4}$ inch) for these units.

EXERCISES

1. Complete: a. 10 quintos = _____ pentos
   b. 15 cincos = _____ quintos
   c. 6 quintos = _____ cincos
   d. 1 pento = _____ cincos
2. Which of the three units is the longest? the shortest?
3. If the measure in pentos is doubled, what happens to the measure in quintos?
4. The two segments shown below have the same length and are measured using another system of measure. (zips and zaps)



a. Which unit is longer?

b. 10 zips = _____ zaps

c. 40 zips = _____ zaps

d. 1 zip = _____ zaps

DISCUSSION QUESTIONS

1. Does using a different unit of measure change the length of a segment? The number used to express the length? Why or why not?
2. Which of three units (pento, quinto, cinco) will give the most precise measurement?

When discussing the system of units in this first section, you may wish to anticipate the exercises of the second half of Lesson 10 by asking the students to analyze the computation they used to obtain their results. If you do so, do not force early generalizations and rob the students of an opportunity to discover the generalizations on their own at a later point in the lesson.

ANSWERS

EXERCISES

1. a. 10 quintos = 2 pentos
   b. 15 cincos = 3 quintos
   c. 6 quintos = 30 cincos
   d. 1 pento = 25 cincos
2. pento; cinco
3. It is doubled.
4. a. zap
   b. 10 zips = 5 zaps
   c. 40 zips = 20 zaps
   d. 1 zip = $\frac{1}{2}$ zaps

DISCUSSION QUESTIONS

1. No. Yes. This highlights the distinction between the length and measure of a segment (see earlier comments on pages T 5 and T 25).
2. Cinco. This a review of the discussion of precision in Lesson 4.

FROM ONE UNIT TO ANOTHER. . .

As soon as you know how two units compare in size, you can convert measures from one unit to another. The previous EXERCISES included some practice in conversion of units. Use the unit relationships in the box to complete TABLE 10-1

1 pento = 5 quintos
1 quinto = 5 cincos
1 ft. = 12 in.
1 yd. = 3 ft.

10 millimeters = 1 centimeter
100 centimeters = 1 meter
1000 meters = 1 kilometer

Complete TABLE 10-1 so the measures in each row represent equivalent lengths. The first two examples are done for you.

TABLE 10-1

	EQUIVALENT LENGTHS		Which unit is longer?
a.	24 inches	$\boxed{2}$ feet	foot
b.	20 quintos	$\boxed{4}$ pentos	pento
c.	☐ quintos	3 pentos	
d.	20 quintos	☐ cincos	
e.	4 feet	☐ inches	
f.	☐ yards	36 feet	
g.	20 mm.	☐ cm.	
h.	3 meters	☐ cm.	
i.	7 cm.	☐ mm.	
j.	☐ km.	4000 m.	

STOP

CHECK YOUR RESULTS BEFORE GOING TO THE NEXT PAGE.

After discussing the first set of **EXERCISES** , have the students complete **TABLE 10-1** .

ANSWERS

TABLE 10-1

c. 15 ; pento
d. 100 ; quinto
e. 48 ; foot
f. 12 ; yard
g. 2 ; cm.
h. 300 ; meter
i. 70 ; cm.
j. 4 ; km.

DO I MULTIPLY OR DIVIDE???

To complete TABLE 10-1, you probably multiplied or divided by some number to get each result. The problem is to decide which operation to use.

FIND A PATTERN . . .

Answer the following questions using the results of TABLE 10-1. (page 70)

1. a. List all examples in which you converted the measure from a smaller to a larger unit. (a is one such example)

   b. When converting from a smaller to a larger unit, which operation (multiplication or division) did you use?

2. a. List all examples in which you converted the measure from a larger to a smaller unit. (c is one such example)

   b. When converting from a larger to a smaller unit, which operation (multiplication or division) did you use?

3. Complete the following statements.

   a. When converting from a smaller unit to a larger unit, ____________ by the number of smaller units in each larger unit.

   b. When converting from a larger unit to a smaller unit, ____________ by the number of smaller units in each larger unit.

FIND A PATTERN gives the student an opportunity to formalize some generalizations concerning his results. It would be useful to discuss the generalizations with the entire class at the end of this lesson or the beginning of the following lesson. Do not insist on memorization but allow tnem to express ideas in their own language. For example, "if you go from a smaller unit to a bigger unit, you divide" would be an acceptable verbalization of conclusion #1.

ANSWERS

1. a. a, b, f, g, j
   b. division

2. a. c, d, e, h, i
   b. multiplication

3. a. divide
   b. multiply

ANOTHER METHOD

Most situations requiring conversion of units can be handled using the guidelines from the previous lesson. However, other methods could be used. Some people have difficulty in determining whether they should divide or multiply.

Another method for conversion of units can be developed using proportions. For example, suppose you needed to convert 54 inches to an equivalent length in feet.

EXAMPLE: 54 in. = ? ft.

SOLUTION:

1. The ratio of feet to inches is $\frac{1}{12}$ as 1 foot = 12 inches.
2. Let x represent the unknown number of feet.
3. The ratio of unknown number of feet to the given number of inches is $\frac{x}{54}$.
4. A proportion you could use to solve this problem is... $\frac{1 \text{ foot}}{12 \text{ inches}} = \frac{x \text{ feet}}{54 \text{ inches}}$

   or

   $$\frac{1}{12} = \frac{x}{54}$$
5. Solve the proportion using the cross-product method.

$$\frac{1}{12} = \frac{x}{54}$$

$$12x = 54$$

$$x = \frac{54}{12} = 4\frac{6}{12} = 4\frac{1}{2}$$

ANSWER: $\boxed{54 \text{ in.} = 4\frac{1}{2} \text{ ft.}}$

The proportion for the example above could also be set up as $\frac{12}{1} = \frac{54}{x}$, using the ratio of inches to feet instead of feet to inches.

NOTE: 12x is an abbreviation for "12 times x".

ANOTHER METHOD

OBJECTIVES

1. Given information such as 1 ft. = 12 in. and 4 ft. = ? in., the student shall be able to...
   a) set up a proportion which could be used to determine the unknown value.
   b) use the cross-product method to solve the proportion.

CONTENT AND APPROACH

This lesson extends the ideas of Lesson 10 by developing the cross-product method as a tool in conversion of units. The development of the lesson is contained in the worked examples. It would be helpful to ask the class to indicate alternate formats for each of the examples. The fact that each problem can be organized in alternate fashion by changing the ratio used is an important concept. Use additional examples if needed.

Some prefer to use as a general method for conversion of units a strategy other than the cross-product. Another alternate strategy is outlined on the remainder of this page and T 72a. If you prefer this method, use it in place of the cross-product for Lesson 11.

Consider the **EXAMPLE** on page 72.

54 in. = ? ft.

Solution: 54 in. are equivalent to 54 x (1 in.)

1 in. is equivalent to $\frac{1}{12}$ ft.

54 in. are equivalent to 54 x ($\frac{1}{12}$ ft.)

are equivalent to 54 x ($\frac{1}{12}$) ft.

are equivalent to $\frac{54}{12}$ ft. = $4\frac{6}{12}$ ft. = $4\frac{1}{2}$ ft.

(Continued on page T 73)

It does not matter which ratio you use as long as you are consistent, using the same ratio on both sides of the proportion.

Before working the exercises, become more familiar with this method of solution by studying the following examples.

Example 1. 3 feet = 1 yard

80 feet = ? yds.

$$\frac{\text{feet}}{\text{yards}} = \frac{3}{1} = \frac{80}{x}$$

$$3x = 80$$

$$x = \frac{80}{3} = 26\frac{2}{3}$$

$$\boxed{80 \text{ feet} = 26\frac{2}{3} \text{ yards}}$$

Example 2. 1 kilometer = .625 miles

60 miles = ? km.

$$\frac{\text{km.}}{\text{mi.}} = \frac{1}{.625} = \frac{x}{60}$$

$$.625x = 60$$

$$x = 60 \div .625$$

$$x = 96$$

$$\begin{array}{r} 96. \\ .625\,\overline{)\,60.000} \\ -5625 \\ 3750 \\ -3750 \end{array}$$

$$\boxed{60 \text{ miles} = 96 \text{ km.}}$$

Example 3. 1 cm. = 10 mm.

86 mm. = ? cm.

$$\frac{\text{cm.}}{\text{mm.}} = \frac{1}{10} = \frac{x}{86}$$

$$10x = 86$$

$$x = 86 \div 10$$

$$x = 8.6$$

$$\boxed{86 \text{ mm.} = 8.6 \text{ cm.}}$$

Other examples can be worked in a similar manner. Some more examples are outlined in the chart below.

Convert	Think	Replace Unit	Result
80 ft. to yds	80 x (1 ft.)	80 x ($\frac{1}{3}$ yd.)	26$\frac{2}{3}$ yd.
60 mi. to km.	60 x (1 mi.)	60 x (1.6 km.)	96 km.
62 m to cm.	62 x (1 m)	62 x (100 cm.)	6200 cm.
73 mm. to cm.	73 x (1 mm.)	73 x (.1 cm.)	7.3 cm.

Regardless of the approach selected, the reasonableness of the result may be checked using the conclusions stated on page 71 (**FIND A PATTERN** , #3).

Example 4. 1 meter = 100 cm.

6.8 meters = ? cm.

$$\frac{m.}{cm.} = \frac{1}{100} = \frac{6.8}{x}$$

$$x = (6.8)\cdot(100)$$

$$x = 680$$

$$\boxed{6.8 \text{ m.} = 680 \text{ cm.}}$$

EXERCISES

1. Complete the following. Use the ratio-proportion method of conversion. Show your work.

   a. 3.4 km. = ______ meters

   b. 65 mm. = _______ meters

   c. 185 cm. = ______ meters

   d. 3.74 m. = ____cm.

   e. 3735 mm. = ____m.

   f. 40 km. = ____miles

2. One centimeter is equivalent in length to about four tenths (.4) of an inch. Assuming 1 cm. = .4 inch, 6 inches = ? cm.

3. One yard is equivalent in length to about nine-tenths (.9) of a meter. Assuming 1 yd. = .9 meter, 4.5 meters = ? yards.

DISCUSSION QUESTION

1. Consider the following statement.

   " I don't see the need for all these different units of of length. All they do is confuse me. Why not have only one or two units of length to measure all distances?"

   Do you agree with this statement? Why or why not?

ANSWERS

EXERCISES

1. a. 3.4 km. = 3400 meters
   b. 65 mm. = .065 meters
   c. 185 cm. = 1.85 m.
   d. 3.74 m. = 374 cm.
   e. 3735 mm. = 3.735 m.
   f. 40 km. = 25 mi.
2. 6 inches = 15 cm.
3. 4.5 meters = 5 yards

DISCUSSION QUESTIONS

Question 1 attempts to motivate a rationale for having several different units of length (The sole reason is not to confuse the student!).

Probably the main reason for several different units is that there are several different magnitudes of length to be measured. Units suitable for measuring short distances are not suitable for long distances (See exercises for Lessons 5 and 12). As increased technology allows previously unmeasurable distances (both short and long) to be measured, new units are created.

Of course, a person is justified in claiming that there are too many units. Many units used in specific situations (chain, furlong, bolt,...) continue to be used out of tradition. In addition, both the metric and English systems are used in the United States when one system would be sufficient.

WHICH IS LONGER?

Make each statement true by inserting > or < . Remember that > means is greater than and < means is less than. You may use your rulers and tables of measures.

1. 1 m. _____ 120 cm.
2. 20 mm. _____ 1 cm.
3. 15 mm. _____ 1 in.
4. 2 cm. _____ 1 in.
5. 3 cm. _____ 1 in.
6. 1 m. _____ 1 yard
7. 1 mile _____ 1 km.
8. 1 cm. _____ 1 in.
9. 5 ft. _____ 62 in.
10. 1.3 cm. _____ 15 mm.
11. Based on your results (questions 4 and 5), what can you conclude concerning the number of centimeters it takes to make a length equal to 1 inch?

ANSWERS

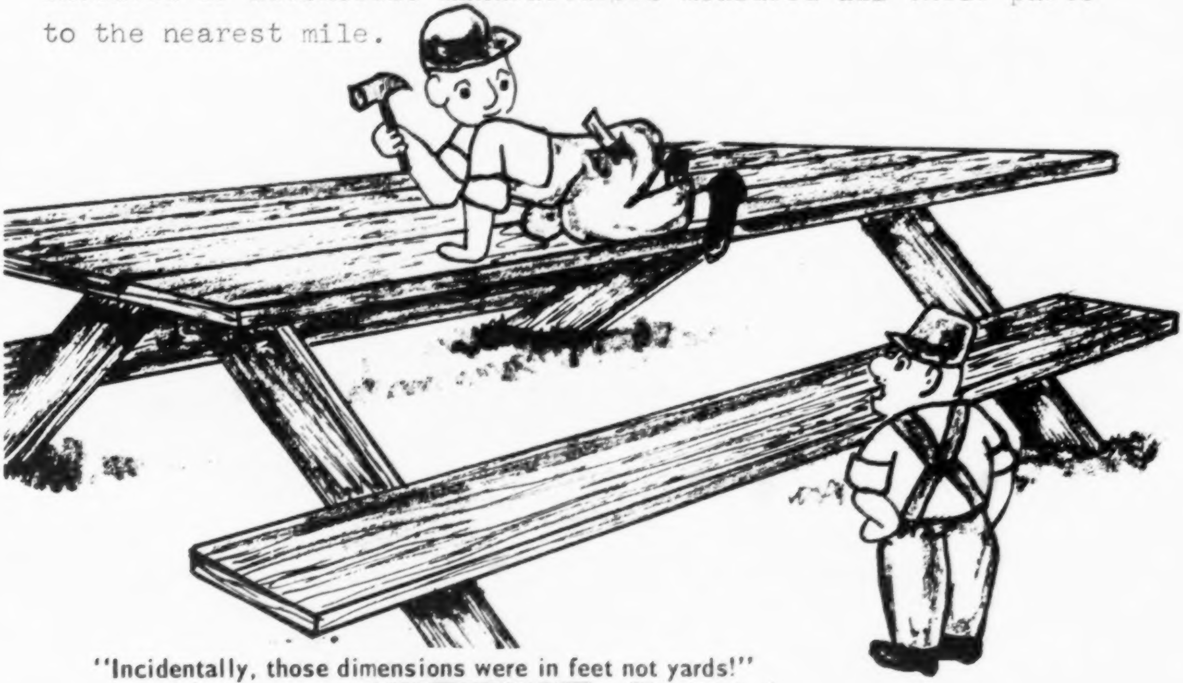
1. 1 m. < 120 cm.
2. 20 mm. > 1 cm.
3. 15 mm. < 1 in.
4. 2 cm. < 1 in.
5. 3 cm. > 1 in.
6. 1 m. > 1 yard
7. 1 mile > 1 km.
8. 1 cm. < 1 in.
9. 5 ft. < 62 in.
10. 1.3 cm. < 15 mm.
11. It takes more than 2 cm. and less than 3 cm. to make 1 inch.

WHAT UNIT SHOULD I CHOOSE???

Before any measurement is made, a person must decide which unit of measure to use. You used several different units of length in Lessons 1-11. The length of the distance to be measured is one factor which will determine the most suitable unit to use.

For example, suppose you measured the distance between Chicago and Detroit. A mileage chart gives this distance as 265 miles. Theoretically, this distance could have been measured using any other English unit of length such as inches, feet, or yards. (265 mi. = 466,400 yds. = 1,399,200 ft. = 16,790,400 in.) However, the mile was selected as the most suitable (or "best") unit to use in this case, partially due to the long distance involved. (A mileage chart in Europe would express a distance as long as the above in kilometers.)

In like manner, a shorter distance or length is usually measured with a unit of shorter length. Imagine the problems involved if automobile manufacturers measured all their parts to the nearest mile.



"Incidentally, those dimensions were in feet not yards!"

WHAT UNIT SHOULD I CHOOSE???

OBJECTIVES

1. Given a distance to be measured, the student will select units of length which are suitable for measuring the given distance.

2. Given a distance to be measured and a list of English and metric units, the student shall select the most suitable (best) English and metric units for measuring the given distance.

3. Provide additional practice in estimating distances.

CONTENT AND APPROACH

The need to select a unit suitable for the distance and purpose of the measurement has been implied and briefly discussed in earlier lessons. For example, see **DISCUSSION QUESTIONS**, p. 12.

In conjunction with the text and cartoon on page 55, the past experiences of the class can be discussed asking questions such as: "Why would you measure the width of your desk in inches (cm.) rather than in yards (meters)?"

The exercises concerning estimation should be conducted in the same spirit as earlier, encouraging the student to make guesses. Some of the exercises can be verified by having on hand samples of the object in question and measuring.

EXERCISES

1. Circle what you would consider the best English unit and the best metric unit for measuring the following. Be able to defend your answer.

a. Length of a river	inch	foot	yard	mile
	centimeter	meter	kilometer	
b. Height of a man	inch	foot	yard	mile
	centimeter	meter	kilometer	
c. Length of a nail	foot	mile	inch	yard
	centimeter	kilometer	meter	
d. Width of a house	inch	yard	mile	foot
	kilometer	meter	centimeter	
e. Distance of a track race	mile	foot	yard	inch
	kilometer	meter	centimeter	

2. List suitable units for the following measurements. (In some instances, more than one unit may be suitable.)

a. The altitude of an airplane . ____________________

b. The depth of an ocean. ____________________

c. The length of an arm. ____________________

d. The width of a picture frame. ____________________

e. The distance from the Earth to the moon. __________

f. The width of a room. ____________________

3. Circle the measurements from each set that are good estimates for each distance. More than one answer from each set may be appropriate.

a. Height of a classroom. (10 ft., 20 ft., $3\frac{1}{3}$ yd., 10 meters)

b. Height of an adult man. (6 ft., 2 meters, 4 yds., 72 in.)

ANSWERS

EXERCISES

1. a. mile; kilometer
   b. foot (and inch); meter (and cm.)
   c. inch; centimeter
   d. foot (and inch); meter (and cm.)
   e. yard (or mile); meter (or km.)

   The above units should be circled for exercise 1.

2. a. miles, km., *feet, *meters
   b. miles, km., *feet, *meters
   c. inches, feet, cm., meters
   d. *inches, *cm.
   e. *miles, *km., A.U. (astronomical unit = 93,000,000 miles)
   f. *feet, *meters, *inches, yards

   *indicates a more probable answer.

3. The following measurements should be circled for exercise 3.
   a. 10 feet, $3 \frac{1}{3}$ yards
   b. 6 feet, 2 meters, 72 inches

4. Make each statement reasonable by filling the blank with a suitable unit of length. (inch, centimeter, foot, meter,...) or its plural.

a. The rails of a railroad track are about 4 ______ part.

b. The height of a telephone pole is about 30 __________.

c. The width of a newspaper column is about 2 __________.

d. The width of a one-dollar bill is about 6.6 __________.

e. The length of a stick of chewing gum is about 8 _____.

f. The height of a dining room table is about $\frac{3}{4}$ of a ____.

5. For each list, arrange the distances in order from shortest to longest. (Indicate the shortest by 1, next shorter by 2,...)

a. 50 meters _____

150 feet _____

Length of an airliner _____

b. 1.1 in. _____

$1\frac{1}{2}$ in. _____

$1\frac{3}{4}$ in. _____

$1\frac{7}{16}$ in. _____

c. 1 meter _____

1 yard _____

$3\frac{1}{2}$ ft. _____

48 in. _____

d. 1 cm. _____

1 in. _____

2 cm. _____

2 in. _____

e. 4 inches _____

$\frac{1}{2}$ meter _____

Length of a one-dollar bill _____

f. $\frac{1}{2}$ mile _____

1000 yd. _____

$\frac{1}{2}$ km. _____

ANSWERS

4. a. feet

b. feet

c. inches

d. centimeters

e. centimeters

f. yard or meter (Meter is the best answer although either answer should be accepted. The normal height of a dining room table is about 30 inches.)

5. a.
50 meters 2
150 feet 1
length of an airliner 3

b.
1.1 in. 1
$1\frac{1}{2}$ in. 3
$1\frac{3}{4}$ in. 4
$1\frac{7}{16}$ in. 2

c.
1 meter 2
1 yard 1
$3\frac{1}{2}$ ft. 3
48 in. 4

d.
1 cm. 1
1 in. 3
2 cm. 2
2 in. 4

e.
4 inches 1
$\frac{1}{2}$ meter 3
length of a one-dollar bill 2

f.
$\frac{1}{2}$ mile 2
1000 yd. 3
$\frac{1}{2}$ km. 1

6. What metric unit of length would you use to measure each of the following?

   a. One lap around a school track. _____

   b. The distance from earth to the nearest star._____

   c. The width of a quarter._____

STOP

CHECK YOUR RESULTS FOR THE EXERCISES
BEFORE GOING TO THE DISCUSSION QUESTIONS.

DISCUSSION QUESTIONS

1. Compare your choices in Exercises 1 and 2 with those of your classmates. Did your choice of unit always agree with the choices of your classmates? Why or why not?

2. How would the purpose a person has in knowing a measurement affect his choice of a unit to use in finding the measure?

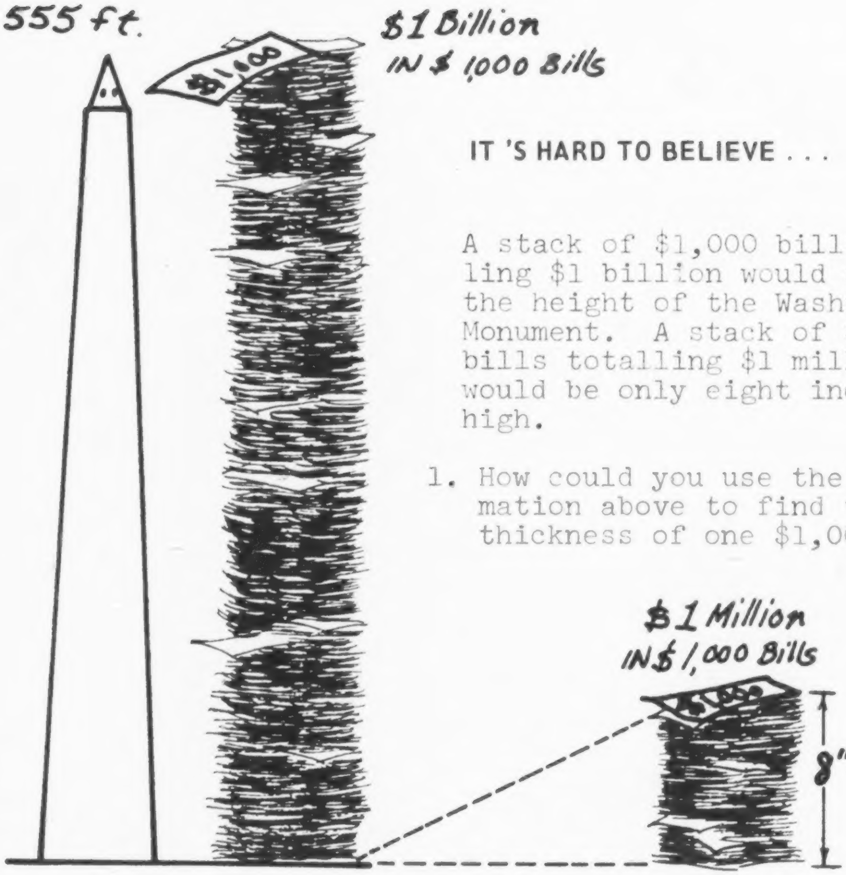
3. Give as many factors as you can which will influence your choice of a suitable unit for a distance to be measured.

ANSWERS

6. a. meter
   b. kilometer
   c. centimeters (or millimeters)

DISCUSSION QUESTIONS

1. Reasons for possible disagreement include those factors discussed in discussion questions 2 and 3.
2. The purpose of the measurement will determine the precision of the measurement and thus affect the choice of unit.
3. Some possible factors are length of the distance, purpose of the measurement (determining the precision), measurement devices available, and which system (English or metric) is being used.



IT'S HARD TO BELIEVE . . .

A stack of $1,000 bills totalling $1 billion would surpass the height of the Washington Monument. A stack of $1,000 bills totalling $1 million would be only eight inches high.

1. How could you use the information above to find the thickness of one $1,000 bill?

2. Describe a procedure for determining the thickness of one sheet of paper or a 3"x 5" card. The two paragraphs above give some clues to a procedure which could be used.

3. Using the above information as a guide, what would be the approximate height of a stack of $1 million in $500 bills? in $100 bills? in $20 bills?

CONTENT AND APPROACH

The use of this page is optional. It does expose the student to a measurement problem and procedure unlike those previously discussed in this booklet.

ANSWERS

1. If one thousand $1000 bills make a stack eight inches high, each bill would be about 8" ÷ 1000 or .008 inches thick.

2. The thickness of one sheet of paper, card, or any thin object of uniform thickness can be obtained by one of the following similar methods.

   a) Measure the total thickness of a stack of 100 objects (or some other convenient number). Divide the total thickness by the number of objects.

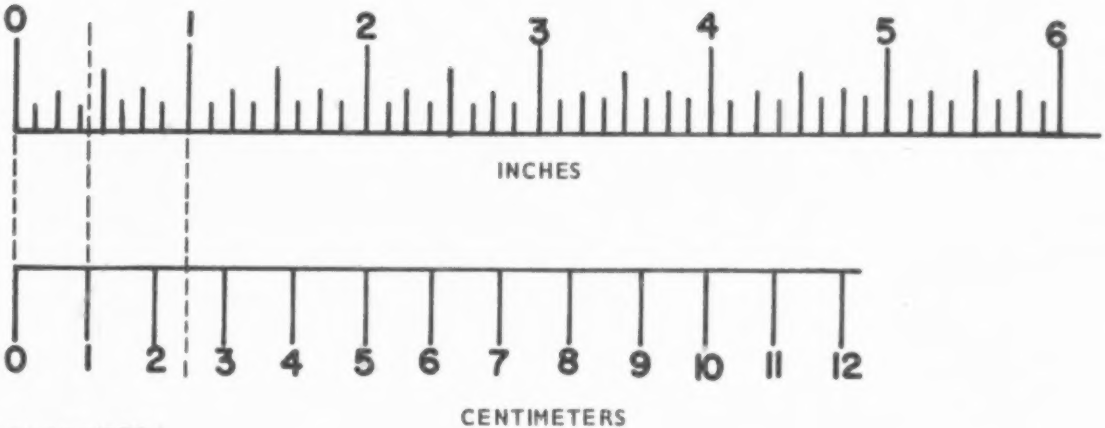
   b) Select some convenient height (1 cm., 1 in.,...). Pile up enough objects to obtain a stack of the selected height. Count the number of objects and divide the height by the number of objects.

3. 16 inches, 80 inches, 400 inches.

SMORGASBORD TIME

The exercises of this section review and extend some of the basic ideas of measurement and estimation discussed in this booklet. Rulers and other linear measuring instruments will be used throughout the remainder of this course. In many situations, you can use your ability to estimate accurately to check the reasonableness of your work.

If you have difficulty with some of these exercises, it may indicate a topic which you should review.



EXERCISES I

1. One inch is equal in length to a little more than 2.54 centimeters. (See above sketch.) One meter is 39.37 inches. Multiply 39.37 by 2.54 and interpret your result.

2. Races in Olympic and international competition are measured in metric units. The Olympic Games include a 100-meter dash rather than a 100-yard dash. Would you expect the record time for 100 meters to be more or less than the record for 100 yards? Why?

SMORGASBORD TIME

OBJECTIVES

1. To review and extend some of the basic ideas of measurement and estimation discussed in Lessons 1-11.

EQUIPMENT AND TEACHING AIDS

STUDENT

1. Piece of string about 8 feet long
2. Yardstick
3. Meter stick
4. Masking tape

*5. Ruler graduated in...

a. .1 inch b. .1 cm. c. $\frac{1}{16}$ inch

CONTENT AND APPROACH

As indicated in the student text, this lesson contains no new material. **EXERCISES I** and **EXERCISES II** require the student to estimate given distances and to judge relative sizes. The emphasis in **EXERCISES III** is on ruler reading skills.

The first two sections are appropriate activities to be completed by teams, the results being discussed with the entire class. The last section should be done as individual work.

Answers to Exercises 1 and 2 are on page T 82.

3. If a runner can run the 100-yard dash in 10 seconds, about how long would you expect him to run the 100-meter dash? (1 m. $\approx$ 1.1 yd.)

4. Express 387 yards in inches; 387 meters in centimeters. Which answer is easier to compute?

5. Insert > or < in the blanks to make each of the following statements true.

   > means "is greater than"

   < means "is less than"

   a. The Saturn V rocket is 362 feet long.
   Ten school busses bumper to bumper _____ the length of the Saturn V rocket.

   b. The scale on a map is 1 inch = 75 miles
   The distance represented by $2\frac{1}{2}$ in. ____ 200 miles.

   c. A railroad box-car is about 45 feet long.
   100 box-cars _____ a mile.

   d. Using special equipment, French explorers recently went down into the ocean to a depth of 100 meters.
   This was _______ a mile.

EXERCISES II

1. Gone to waist...

   For this problem you will be given a piece of string. Lay the string on a flat surface and pull the two ends in the direction of the arrows until the distance around the loop equals the distance around your waist.
   (See page 62.)

ANSWERS

EXERCISES I (pp. 81-82)

1. 39.37 x 2.54 = 99.9998
   One meter = 100 centimeters

2. More--100 meters is longer than 100 yards.
   An almanac will give specific data on world records.
   (See **THINGS TO DISCUSS**.)

3. 11 sec.

4. 387 yd. = 13,932 in.
   387 m. = 38,700 mm.
   The second answer is easier to compute.

5. a. >   c. <
   b. <   d. <

THINGS TO DISCUSS

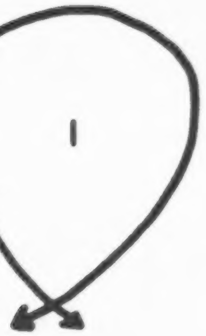
EXERCISES I (pp. 81-82)

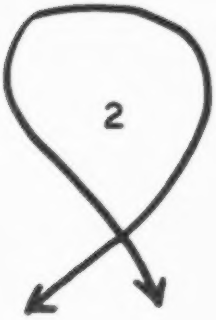
2. What Olympic events are comparable to the following events?
   - a. 220 yd. dash   b. 440 yd. dash
   - c. 880 yd. run   d. mile run

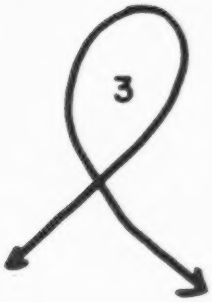
   (Ans a. 200 m.   b. 400 m.
   c. 800 m.   d. 1500 m.)

   How will the times for these races compare with their metric counterparts?

4. Why is computation in the metric system easier?







Check your estimate by wrapping the loop around your waist. How close was your estimate to your actual waist measurement in inches? in centimeters?

2. A fish story...

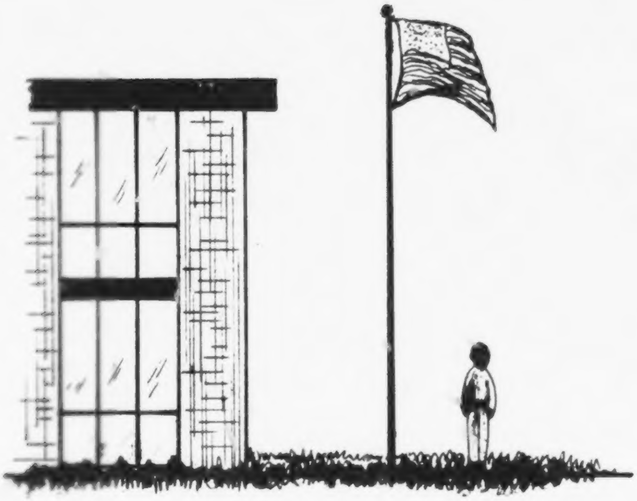
   Stu caught a 22 inch pike while ice fishing. Demonstrate the length of the fish by holding your hands 22 inches apart. Try this once with your eyes closed. Have a classmate check each estimate by measuring the distance between your hands.

   Questions 3 and 4 are related to the following story and cartoon. (See page 63.)

   Estimates can be made in a variety of ways. The following story illustrates one method used to obtain an estimate.

THINGS TO DISCUSS

1. Most people are "way off" in their estimate because they visualize their waist as circular rather than elliptical. Answers will vary.



Terry wished to estimate the height of the school flagpole. (Why? I don't know. Let's assume he had a good reason.) How could he get an estimate? One method would be to reason as follows.

> "Each floor of the school is about 10 ft. high, making the school building about 20 ft. high. The flagpole is about a half-floor taller than the school building. Therefore, the flagpole is about 25 ft. high."

3. What are some of the factors which will determine the distance that is used as a guide in estimating?

4. Find some other method for estimating the height of the flagpole.

ANSWERS

*3. Objects in the immediate environment--what lengths are known--distances that are approximately the same length as that being estimated.

4. Answers will vary.

* These responses are to be used as a guide. Answers may vary among members of the class.

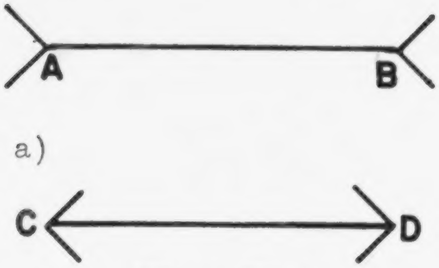
5. Estimate each of the following lengths by placing two pieces of tape that distance apart on a flat surface. (desk, floor,...) Check your estimates by measurement and record your results.

a. 10 feet c. 8 inches

b. 5 meters d. 30 centimeters

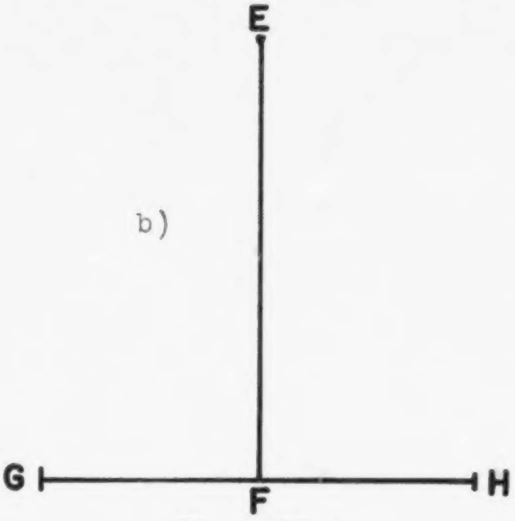
6. Estimate which of each pair of segments is longer. Check your estimates by measurement.

a)



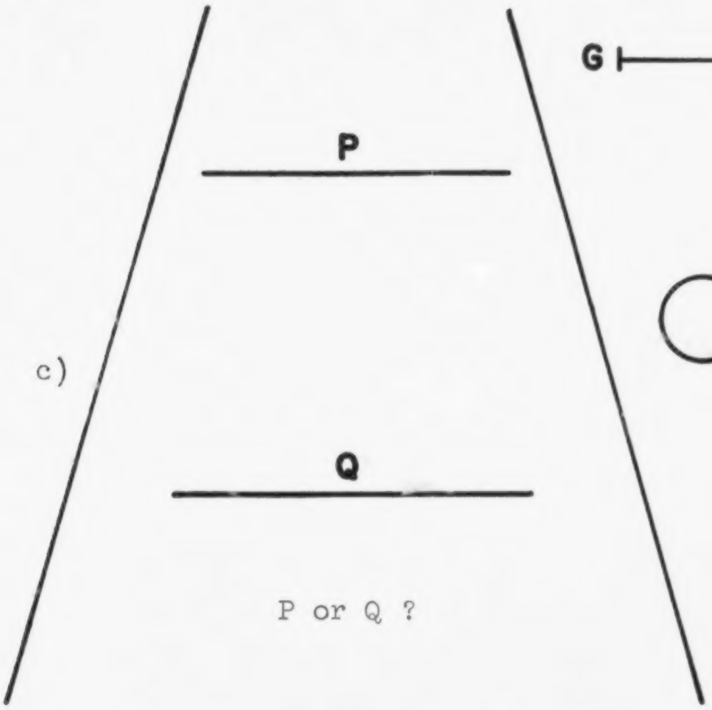
$\overline{AB}$ or $\overline{CD}$?

b)



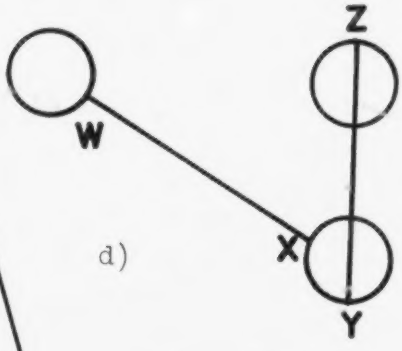
$\overline{EF}$ or $\overline{GH}$?

c)



P or Q ?

d)



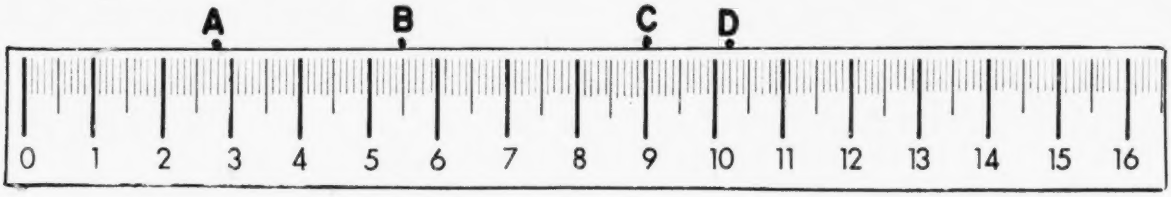
$\overline{WX}$ or $\overline{YZ}$?

6. With the exception of (c), all pairs of segments are the same length. In (c), P is $1\frac{13}{16}$ in. long. (4.5 cm.) and Q is $2\frac{1}{8}$ in. long. (5.3 cm.)

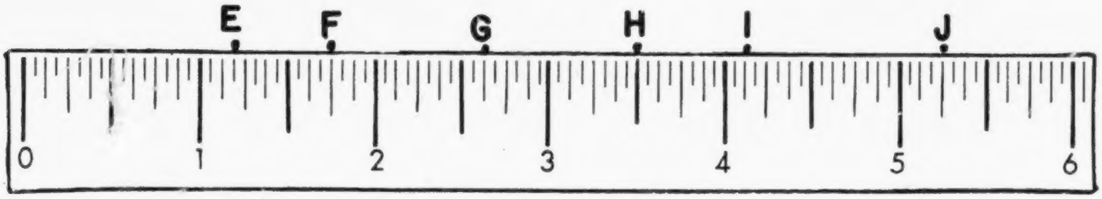
EXERCISES III

1. Give the distance from 0 for each point indicated. Ruler I is graduated in centimeters. Rulers II and III are graduated in inches.

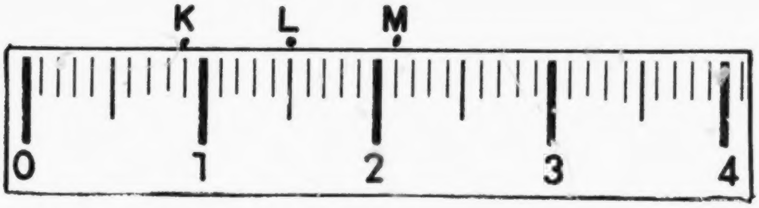
I


A
B
C
D
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

II


E
F
G
H
I
J
0
1
2
3
4
5
6

III



2. Using the rulers pictured above, give the distance between the following pairs of points.

a. B and C	c. G and H
b. F and H	d. L and M

3. For each ruler location, give the whole number measure to which it is closest. Part a is done for you.

a. $1\frac{5}{16}$ in. 1 in.

b. 8.7 cm. _____ cm.

c. $4\frac{5}{8}$ in. _____ in.

d. $2\frac{1}{4}$ in. _____ in.

ANSWERS

1. A. 1.8 cm.

   B. 5.5 cm.

   C. 9.0 cm.

   D. 10.2 cm.

   E. $1\frac{3}{16}$ in.

   F. $1\frac{12}{16}$ or $1\frac{3}{4}$ in.

   G. $2\frac{10}{16}$ or $2\frac{5}{8}$ in.

   H. $3\frac{8}{16}$ or $3\frac{1}{2}$ in.

   I. $4\frac{2}{16}$ or $4\frac{1}{8}$ in.

   J. $5\frac{4}{16}$ or $5\frac{1}{4}$ in.

   K. .9 in.

   L. 1.5 in.

   M. 2.1 in.

The precisions desired for exercise #1 are not indicated. Use the following precisions.

PROBLEM	PRECISION
A, B, C, D	nearest .1 cm.
E, F, G, H, I, J	nearest $\frac{1}{16}$ in.
K, L, M	nearest .1 in.

2. a. 3.5 cm

   b. $1\frac{3}{4}$ in.

   c. $\frac{7}{8}$ in.

   d. .6 in.

3. a. 1 in.

   b. 9 cm.

   c. 5 in.

   d. 2 in.

THINGS TO DISCUSS

1. If practice is needed, vary this exercise by asking for the distances to the nearest whole, $\frac{1}{2}$, $\frac{1}{4}$, ... in. (cm.)

4. Indicate by a letter and arrow each of the following locations on the rulers below. Ruler I is graduated in centimeters. Ruler II is graduated in inches.

A. 6.8 cm.

B. 4.5 cm.

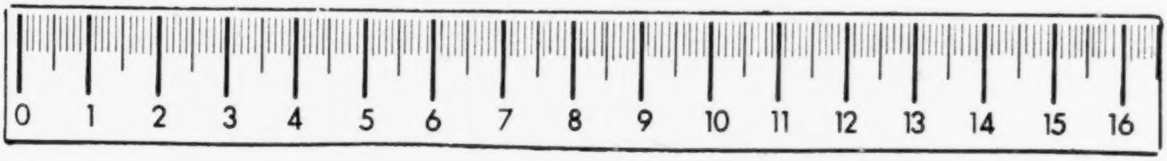
C. $3\frac{7}{8}$ in.

D. $4\frac{3}{4}$ in.

E. $5\frac{9}{16}$ in.

F. $\frac{5}{8}$ in.

I


0
1
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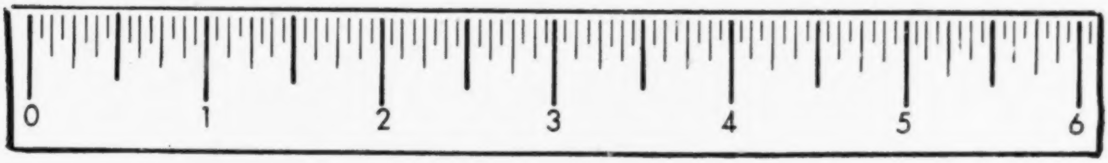
16

II



5. Draw segments of the following lengths.

A. 8.7 in.

B. $6\frac{1}{8}$ in.

C. 10.4 cm.

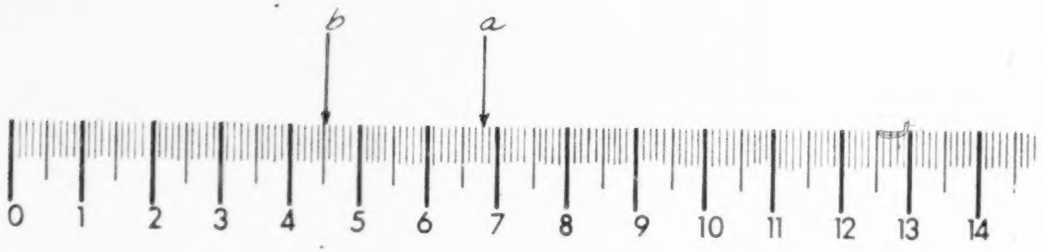
D. $4\frac{5}{16}$ in.

E. $\frac{1}{2}$ in.

F. $4\frac{1}{4}$ in.

ANSWERS

4.


b
a
0
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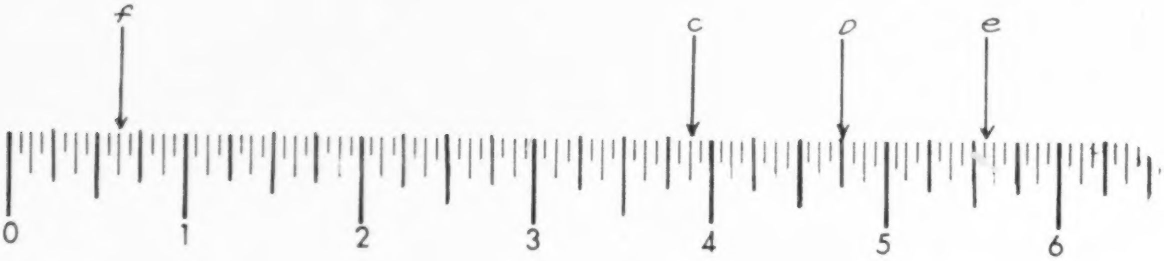

f
c
d
e
0
1
2
3
4
5
6

5. These must be checked by measuring. An overlay answer sheet can be made using a thin sheet of paper, acetate sheet, or sheet of tracing paper.

THINGS TO DISCUSS

5. Will your segments be exactly the same length as those of your classmates? (No) Why or why not?

This illustrates the approximate nature of measurement.

POINT

Use the following questions and problems to check your understanding of Lessons 10-12. If you have difficulty with any questions, it may indicate a topic you should review.

1. Complete the following.

   a. 2.8 m. = ____ cm.

   b. 87 mm. = ____ cm.

   c. $1\frac{3}{4}$ ft. = ____ in.

   d. 6 yd. = ____ ft.

   e. 85 cm. = ____ mm.

   f. 27 in. = ____ ft.

   g. 85 cm. = ____ m.

   h. 2.1 km = ____ m.

2. Suppose that 2 sticks = 5 logs, where sticks and logs are units of length.

   a. Which unit is longer?

   b. 15 logs = __?__ sticks.

3. For each list, arrange the distances in order from shortest to longest. (Indicate the shortest by 1, next shorter by 2,...)

   a. 2 meters ______
   2 yards ______
   2 kilometers ______

   b. 3 meters ______
   8 feet ______
   height of an adult man ______

   c $3\frac{11}{16}$ inches ______
   $3\frac{3}{4}$ inches ______
   $3\frac{5}{8}$ inches ______

   d. 1 kilometer ______
   1 mile ______
   1 centimeter ______
   1 inch ______

4. Given a distance to be measured you should be able to choose a English and a metric unit that would be a suitable unit for measuring the distance. (See Lesson 12.)

ANSWERS

1. a. 280 cm.
   b. 8.7 cm.
   c. 21 in.
   d. 18 ft.
   e. 850 mm.
   f. $2\frac{1}{4}$ ft.
   g. .85 m.
   h. 2100 m.

2. a. The stick is the longer unit.
   b. 15 logs = 6 sticks.

3. a. 2 meters 2
   2 yards 1
   2 kilometers 3

   b. 3 meters 3
   8 feet 2
   height of an adult man 1

   c. $3\frac{11}{16}$ inches 2
   $3\frac{3}{4}$ inches 3
   $3\frac{5}{8}$ inches 1

   d. 1 kilometer 3
   1 mile 4
   1 centimeter 1
   1 inch 2

4. No response is intended. This statement is to be used as a self-check by the student.

AND IN CONCLUSION . . .

This booklet concentrated on the measurement of length. The standard units for measuring the lengths of common objects are well known. A person's height is measured in feet and/or inches. Lengths of smaller objects are given in inches or centimeters (or fractions of these units). Longer distances are measured in miles or kilometers.

However, this is not the final word on length. Just as the caveman invented units of length for his use (hand, span, cubit,...), new units of length are being invented today. In this booklet, measurements were made to the nearest $\frac{1}{16}$ in. and .1 cm. However, measurements are being made by scientists to the nearest billionth of an inch! To make measurements this precise, new measuring instruments and units have been invented.

Since there is no such thing as man-made perfection (despite the claims of many advertisements), there will never be any truly exact measurements. However, measurements and instruments will continue to be invented and refined. Lengths which are unmeasurable today will become measurable in the future.

Many of these measures and instruments are used only by persons in special fields of science and industry. It is not necessary that anyone know about all of them. It is important, however, that you be aware of their existence and what implications they hold for the future.

Future booklets and lessons concerning linear measurement will expose you to some of these units (their definition and use) and indicate instruments and methods that are used to obtain these precise measurements.

APPENDICES

A. CHOOSE THE BEST ESTIMATE

B. INVITATION TO PROBLEM SOLVING

EXPERIMENTAL ERROR

C. RULERS WITH GAPS

CHOOSE THE BEST ESTIMATE

In this booklet the closeness of your estimates was checked by (1) measuring the distance and (2) computing the difference between the measured distance and the estimate.

Consider the following information.

ESTIMATE	MEASURED LENGTH	DIFFERENCE
A. 30 ft.	29 ft. 4 in.	8 in.
B. 9 in.	10 in.	1 in.
C. 30 cm.	28 cm.	2 cm.

1. Which estimate is closest to the measured length?
2. Which estimate best represents the length being estimated?
3. Which of the three estimates is the best? Why?

NOTE: These or similar questions may be reproduced on a ditto master for distribution to students if this activity is used.

CHOOSE THE BEST ESTIMATE

CONTENT AND APPROACH

Data such as that on page 90 can be used to indicate that the accuracy of an estimate depends not only on its closeness to the measured length but also on the relative sizes of the difference and the measured length.

Given several different lengths and estimates for these lengths, determining the best estimate requires more than answering, "Which estimate is closest to its measured length?". A more appropriate question would be, "Which estimate best represents its measured length?". Answering this question involves comparing the ratios of the difference to the measured length for each situation. The smaller the ratio, the better the estimate represents the measured length.

ANSWERS

1. Estimate B (9 in.)
2. Estimate A (30 ft.)
3. Estimate A - this estimate best represents its measured length (see above comments.).

Of course, in some situation a small range of error is not enough to guarantee a good estimate. You do not want less concrete than needed or too little money in a budget. This aspect of estimation and a more formal treatment of topics such as relative error, accuracy, per cent of error, and precision will be developed in later booklets.

INVITATION TO PROBLEM SOLVING
EXPERIMENTAL ERROR

OBJECTIVES

This activity is set up for use as a class discussion project. The material introduces the notion of experimental error. The following points are highlighted by the questions posed.

1. Experimental error is inevitable; it can be reduced but not eliminated.
2. Since error is inevitable, data obtained from experiments is almost always subject to several possible interpretations.
3. One person's "experimental error" may be the basis of another person's research. The variation of data may not have been due to entirely to experimental error but a result of improper measuring procedures.

CONTENT AND APPROACH

A suggested procedure for the use of the material is as follows. The teacher presents the background information and data to the students (via blackboard, overhead projector, or ditto sheet). The teacher then poses the problems and questions in the sequence. If an earlier response answers a later question, this question may either be deleted or used as a review question when it appears in the sequence of questions to be asked.

The sections labeled TO THE STUDENT contain, in sequence, the questions to be posed to the student. The sections labeled TO THE TEACHER contain comments for the teacher, indicating the purpose and content of the material, providing examples of possible answers, and an indication of what comes next in the sequence. If an overhead projector is used to present the material, transparencies summarizing the data and questions to be posed could be used.

All sections (in particular, sections E and F) may not be appropriate for your students. The material is designed to be used in sequence. However, the point at which you choose to discontinue the discussion is optional. For example, you may choose to use only sections A-D.

The data provided is hypothetical. This activity may be more relevant if a similar experiment with a metal bar or similar object were actually conducted, using the data collected as the basis for discussion.

A. TO THE STUDENT

Over a period of a week, four students made one measurement each day of a metal bar. They were given the impression that each was measuring a different bar each time. Their measurements (to the nearest tenth of a millimeter) were as follows:

	Student 1	Student 2	Student 3	Student 4
Measurement 1	500.0	500.0	499.8	500.1
2	499.9	500.0	499.9	500.1
3	500.0	500.0	500.0	500.2
4	500.1	500.0	500.1	500.2
5	500.1	500.0	500.1	500.3

1. Suppose an estimate of the length of the bar was required to be used in many arithmetic calculations. In that case, what estimate would you choose as being "good" enough and also convenient?
2. If a future experiment required the best estimate of the length of the bar what number would you choose to use and how would you go about calculating it?

B. TO THE STUDENT

Notice the difference between the measurements reported by Student 2 and those reported by Students 1,3, and 4.

1. How would you describe the difference?
2. What do you think explains the difference?

A. TO THE TEACHER

The two questions are put together so that the students will have a chance to contrast the idea of a "convenient" measure with the idea of a "best" measure. The most convenient measure would, of course, be 500 mm. One "best" estimate, making no further study of differences between early and late measures and the measures by different students, would be a simple arithmetic mean or average.

B. TO THE TEACHER

The difference between the report of Student 2 and the others is the uniformity of Student 2's measurements. Some students may explain this as due to greater accuracy on the part of Student 2. Some students may suggest the contrary--that Student 2 let his first measurement influence his reasing of later measurements (or even that Student 2 is cheating in an extremely silly way).

A person competent in measurement should obtain measurements that are close together (within a small interval). However, complete uniformity (where measurements are all the same) is not easily achieved. Thus, results such as those obtained by Student 2 are usually suspect. This section could end by introducing the term "experimental error" as meaning unavoidable inaccuracy and inconsistency of measurement.

C. TO THE STUDENT

Now compare the measurements reported by STUDENT 4 with the other reports.

1. What overall difference is there?
2. How would you explain this difference?

Hint: Think of the measurement you would get if you read the right end of the measuring stick while standing at its left end, the stick being on top of the measured bar. Then think of what the measurements might seem if you stood in line with the right end of the measuring stick.

D. TO THE STUDENT

Now inspect each column of measurements from the top down.

1. What trend is noticeable in the measurements for each of the students?
2. What--besides a new kind of systematic error--might explain this trend?

Hint: The bar being measured was made of metal. Assume that the measurements were made in the late spring in the Middle West.

C. TO THE TEACHER

Student 4's measures are uniformly higher than most. The hint is intended to lead the student to see that conditions of measurement may vary from one person to another and thus lead to different data. If you feel that your class is ready for it you can discuss the distinction between random error and systematic error. Student 4 represents a case of systematic error. The entire group of measurements, irrespective of students, would come close to exemplifying random error. The most important reason for making the distinction between random and systematic error is that our common ways of making use of numerous measurements (by calculating the arithmetic mean, the mode, or the median) are ways of correcting random error. We have no readily available way of detecting or correcting systematic error except by watching our habits. Hence, systematic error is potentially a source of danger where a reasonable amount of random error is not.

D. TO THE TEACHER

There is a clearly noticeable trend (ignoring Student 2) toward larger measurements on the later days. The hint--which is weak--is intended to turn the students' attention to the possibility that the bar actually got longer--expanded-- as the days grew warmer. You will probably need to guide the students to see and understand this possibility.

E. TO THE STUDENT

Suppose that the trend just discussed (Part D) had been overlooked or treated as experimental error. Further suppose that the measurements were serious ones used to support a certain theory according to which the rod ought to be nearer 500.0 mm long than 500.1 mm. Unless this experiment were repeated by others at different times and places, what might happen in the science which included this theory?

F. TO THE STUDENT

In view of what your answer in Part D indicates, explain what is meant by saying that science is a social enterprise.

E. TO THE TEACHER

The point of this was stated in the **OBJECTIVES** (#3, page T 91). What one man ascribes to experimental error may be the basis, through research, of other and more useful investigations.

F. TO THE TEACHER

The point is that for some problems many heads are better than one; that science depends on debates and differences, on alternative approaches to problems.

RULERS WITH GAPS

This series of cards outlines an activity concerning the measurement of length.

INSTRUCTIONS

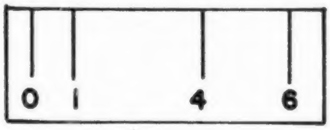
1. Read the directions for each question carefully before answering the question.
2. Write your answers on the Student Report Sheet. Do not write on the cards.
3. When you have finished, check your answers. If after careful study, you still do not understand any question, ask for help.

CARD #1

EQUIPMENT INVENTORY: 3 strips of cardboard, ruler or yardstick, scotch tape.

ACTIVITY:

PART I



1. Make a 6-inch ruler by labeling the zero point near the left end of a cardboard strip and 3 other points, at 1 inch, 4 inches, and 6 inches from zero.

2. Name two points on the ruler scale above such that the distance between them could be used to measure a length of 1 inch; 2 inches; 3 inches; 4 inches; 5 inches; 6 inches. (The first two parts are done as an example.)

RULERS WITH GAPS

CONTENT AND APPROACH

This activity is designed as an enrichment project to be done by either individual or small groups (2-3) of students. The instructions and questions for students are printed on cards. Students are instructed to write their results on the **STUDENT REPORT SHEET** which are provided.

It would be difficult to confine this activity to a single class period. Some of the questions are open-ended (for example, card # 6). You may wish to give this project to interested students to be completed over a period of 2-3 days or a week. It would be helpful to follow up this activity by discussing the results with those students who attempt it.

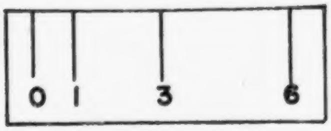
EQUIPMENT AND TEACHING AIDS

STUDENT

* 1. Activity cards (copy reproduced on pp. 96-99)
2. Three strips of cardboard (each at least 6 in. long)
3. Scotch tape
4. Ruler or yardstick
* 5. Student Report Sheet

CARD #2

3. Suppose a 6-inch ruler was labeled like the one below.



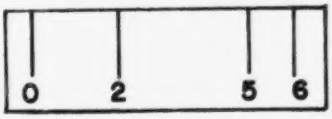
a. Using only the 4 points shown above, could all the whole number lengths from 1 through 6 (1,2,3,4,5,6) be measured? If not, which one(s) could not be measured?

b. Could some lengths be measured in more than one way? If so, indicate how this could be done.

c. Try some other combinations of 4 points (including 0 and 6) on a 6-inch ruler. Are there other combinations which will enable you to measure directly all the whole number lengths from 1 through 6?

CARD #3

4. Consider a 6-inch ruler labeled like the one below.



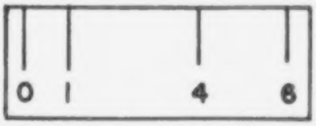
a. In what ways does the ruler above differ from the ruler in #1?

b. In what ways is this ruler like the ruler in #1?

CONTENT AND APPROACH

PART I (Cards 1-3)

To measure directly any whole number of inches from 1 to 6 with a 6-inch stick, it is necessary to label only 4 points: the endpoints (0 and 6) and two points at 1 inch and 4 inches from one of the endpoints.



For example, consider the 6-inch ruler pictured above. From these 4 points, a pair of points to be used for measuring can be chosen in $\frac{1}{2} \times 4 \times 3 = 6$ ways (1 of the points can be chosen in 4 ways, and then the other in 3 ways, so an ordered pair of points can be chosen in 4×3 ways; but the order does not matter, so the number of distinct pairs of points that can be chosen is $\frac{1}{2} \times 4 \times 3$). Using 4 points, therefore, the greatest number of different distances that could be measured is 6. The 4 points marked above do enable 6 different distances to be measured, for every pair corresponds to a different distance: $1 - 0 = 1$, $6 - 4 = 2$, $4 - 1 = 3$, $4 - 0 = 4$, $6 - 1 = 5$, and $6 - 0 = 6$.

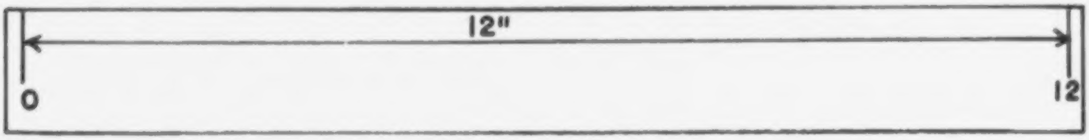
It would not be possible to measure every whole number of inches from 1 to 6 with fewer points. With 1 extra point besides the ends (i.e. 3 points including the ends), for example, the greatest number of distances that could be measured would be $\frac{1}{2} \times 3 \times 2 = 3$.

It is not true that any 4 points would enable 6 different distances to be measured. For example, if 0,1,3,and 6 were marked, a length of 3 inches could be measured in 2 different ways($3 - 0$ and $6 - 3$), and 4 inches could not be measured at all. Are there any points other than those at 1 inch and 4 inches from one end that will suffice?

CARD #4

PART II

1. Tape two of the cardboard strips together so that you have a length at least 12 inches long.
2. Label a zero point near the left end of the strip and a point near the right end that is 12 inches from zero.

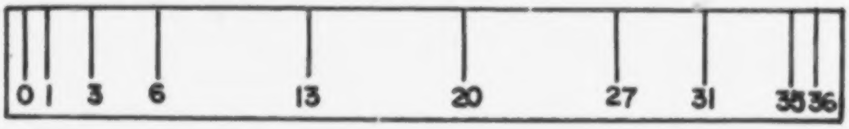


3. Label four other points on this 12 inches so that all the whole number lengths from 1 through 12 could be measured directly using only the six labeled points. (There is more than one solution.)

CARD #5

PART III

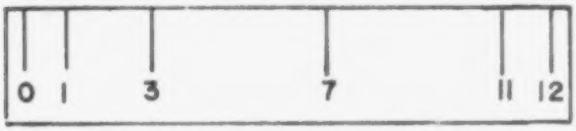
1. A yardstick has been labeled like the one pictured below.



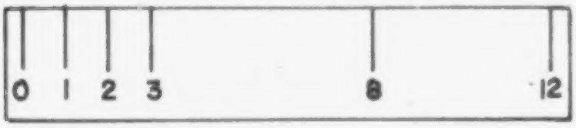
This yardstick can be used to measure directly any whole number length of inches from 1 to 36. For example, 17 inches is given by the distance from 3 to 20, and 30 inches by the distance from 1 to 31 or from 6 to 36. Complete the table given on the **STUDENT REPORT SHEET** .

PART II (Card 4)

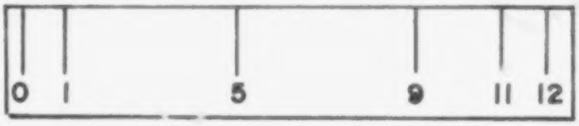
Consider now a stick 12 inches long. With 5 points, including the ends, marked, the greatest number of different distances that could be measured is $\frac{1}{2} \times 5 \times 4 = 10$, and so 5 points are not enough to measure every whole number of inches from 1 to 12. With 6 points (i.e. 4 besides the ends) marked, the greatest number of different distances is $\frac{1}{2} \times 6 \times 5 = 15$, so it would seem likely that 4 extra points would suffice, perhaps in several different ways. A few trials will indicate that one solution is:



Another quite different solution is



the next diagram



also shows a solution, but it is not essentially different from the first one, because the distances of the marked points from the right-hand end of the ruler are the same as the distances of the marked points from the left-hand end in the first solution.

TO EXPLORE...

1. Find another solution such as the one on Card #5 using 10 points. (including 0 and 36)
2. Can you find a solution using only 9 points? (including 0 and 36) If so, demonstrate your solution.

PART III (Card 5)

The yard-stick pictured on the card can be used to measure directly any whole number of inches from 1 to 36. For example, 17 is given by the interval (3,20), 21 by the interval (6,27), 30 by the interval (1,31) or by (6,36).

With 10 marked points, (8 together with the ends), the greatest number of different distances is $\frac{1}{2}$ x 10 x 9 = 45; in fact, with the 10 points that are marked, only 36 different distances can be measured because some can be measured in more than one way. With only 9 points marked, i.e. 7 together with the ends, the greatest number of different distances is $\frac{1}{2}$ x 9 x 8 = 36, which is just the number required. Is it in fact possible to mark 7 points on the ruler (besides the ends) so that all 36 pairs determine different distances? (i.e., can a solution be found with one fewer point marked than in the solution given?).

APPENDIX C - STUDENT REPORT SHEET

NAME(S)____________________ DATE____________________

PART I

	Distance Measured	Points on Scale
2.	1 inch	0,1
	2 inches	4,6
	3 inches	
	4 inches	
	5 inches	
	6 inches	

3. a.

b.

c.

4. a.

b.

PART III

	Distance Measured	Points on Scale
1.	1 inch	0,1 or 35,36
	2 inches	
	3 inches	

For a discussion of some possible solutions to Part II (Card#4) see CONTENT AND APPROACH (Part I).

APPENDIX C - STUDENT REPORT SHEET

NAME(S) Answer Key DATE ____________

PART I

	Distance Measured	Points on Scale
2.	1 inch	0,1
	2 inches	4,6
	3 inches	1,4
	4 inches	0,4
	5 inches	1,6
	6 inches	0,6

3. a. No- A length of 4 cannot be measured using the points on the ruler shown in # 3.

   b. Yes- A length of 3 can be measured by the distance between 0,3 or 3,6.

   c. Yes- Using the points 0,2,5,6.

4. a The points labeled are different.

   b. The middle pts. [(1,4) and (2,5)] are both 3 inches apart- 1 pt. of each pair is 1 inch from one end, the other pt, is 2 inches from the other end. Both rulers can be used to measure all whole numbers from 1 through 6.

PART III

	Distance Measured	Points on Scale
1.	1 inch	0,1 or 35,36
	2 inches	1,3
	3 inches	3,6 or 0,3

Continued

Distance Measured	Points on Scale
4 inches	
5 inches	
6 inches	
7 inches	
8 inches	
9 inches	
10 inches	
11 inches	
12 inches	
13 inches	
14 inches	
15 inches	
16 inches	
17 inches	3,20
18 inches	
19 inches	
20 inches	
21 inches	
22 inches	
23 inches	
24 inches	
25 inches	
26 inches	
27 inches	
28 inches	
29 inches	
30 inches	1,31 or 6,36
31 inches	
32 inches	
33 inches	
34 inches	
35 inches	
36 inches	0,36

Continued

Distance Measured	Points on Scale
4 inches	27,31 or 31,35
5 inches	1,6 or 31,36
6 inches	0,6
7 inches	6,13 or 13,20 or 20,27
8 inches	27,35
9 inches	27,36
10 inches	3,13
11 inches	20,31
12 inches	1,13
13 inches	0,13
14 inches	6,20 or 13,27
15 inches	20,35
16 inches	20,36
17 inches	3,20
18 inches	13,31
19 inches	1,20
20 inches	0,20
21 inches	6,27
22 inches	13,35
23 inches	13,36
24 inches	3,27
25 inches	6,31
26 inches	1,27
27 inches	0,27
28 inches	3,31
29 inches	6,35
30 inches	1,31 or 6,36
31 inches	0,31
32 inches	3,35
33 inches	3,36
34 inches	1,35
35 inches	0,35 or 1,36
36 inches	0,36